

Trigonometry Exercise No. 2

Question 1:

Evaluate the following

(i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

(iii) $\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

(iv) $\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

(v) $\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

Answer :

(i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

$$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2 + \frac{3}{4} - \frac{3}{4} = 2$$

(iii) $\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\frac{2}{\sqrt{3}}} + 2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2 + 2\sqrt{3}}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}}{\sqrt{2}(2 + 2\sqrt{3})} = \frac{\sqrt{3}}{2\sqrt{2} + 2\sqrt{6}}$$

$$= \frac{\sqrt{3}(2\sqrt{6} - 2\sqrt{2})}{(2\sqrt{6} + 2\sqrt{2})(2\sqrt{6} - 2\sqrt{2})}$$

$$= \frac{2\sqrt{3}(\sqrt{6} - \sqrt{2})}{(2\sqrt{6})^2 - (2\sqrt{2})^2} = \frac{2\sqrt{3}(\sqrt{6} - \sqrt{2})}{24 - 8} = \frac{2\sqrt{3}(\sqrt{6} - \sqrt{2})}{16}$$

$$= \frac{\sqrt{18} - \sqrt{6}}{8} = \frac{3\sqrt{2} - \sqrt{6}}{8}$$

(iv) $\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

$$= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1} = \frac{\frac{3}{2} - \frac{2}{\sqrt{3}}}{\frac{3}{2} + \frac{2}{\sqrt{3}}}$$

$$= \frac{\frac{3\sqrt{3}-4}{2\sqrt{3}}}{\frac{3\sqrt{3}+4}{2\sqrt{3}}} = \frac{(3\sqrt{3}-4)}{(3\sqrt{3}+4)}$$

$$= \frac{(3\sqrt{3}-4)(3\sqrt{3}-4)}{(3\sqrt{3}+4)(3\sqrt{3}-4)} = \frac{(3\sqrt{3}-4)^2}{(3\sqrt{3})^2 - (4)^2}$$

$$= \frac{27+16-24\sqrt{3}}{27-16} = \frac{43-24\sqrt{3}}{11}$$

$$(v) \frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{5\left(\frac{1}{4}\right) + \left(\frac{16}{3}\right) - 1}{\frac{1}{4} + \frac{3}{4}}$$

$$= \frac{\frac{15+64-12}{12}}{\frac{4}{4}} = \frac{67}{12}$$

Question 2:

Choose the correct option and justify your choice.

$$(i) \frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} =$$

- (A). $\sin 60^\circ$
- (B). $\cos 60^\circ$
- (C). $\tan 60^\circ$
- (D). $\sin 30^\circ$

$$(ii) \frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} =$$

- (A). $\tan 90^\circ$
- (B). 1
- (C). $\sin 45^\circ$
- (D). 0

$$(iii) \sin 2A = 2\sin A \text{ is true when } A =$$

- (A). 0°
- (B). 30°
- (C). 45°
- (D). 60°

$$(iv) \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} =$$

- (A). $\cos 60^\circ$
- (B). $\sin 60^\circ$

(C). $\tan 60^\circ$

(D). $\sin 30^\circ$

Answer :

$$\begin{aligned} \text{(i)} \quad & \frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} \\ &= \frac{2 \left(\frac{1}{\sqrt{3}} \right)}{1 + \left(\frac{1}{\sqrt{3}} \right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} \\ &= \frac{6}{4\sqrt{3}} = \frac{\sqrt{3}}{2} \end{aligned}$$

Out of the given alternatives, only $\sin 60^\circ = \frac{\sqrt{3}}{2}$

Hence, (A) is correct.

$$\begin{aligned} \text{(ii)} \quad & \frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} \\ &= \frac{1 - (1)^2}{1 + (1)^2} = \frac{1 - 1}{1 + 1} = \frac{0}{2} = 0 \end{aligned}$$

Hence, (D) is correct.

(iii) Out of the given alternatives, only $A = 0^\circ$ is correct.

As $\sin 2A = \sin 0^\circ = 0$

$2 \sin A = 2 \sin 0^\circ = 2(0) = 0$

Hence, (A) is correct.

$$\begin{aligned} \text{(iv)} \quad & \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} \\ &= \frac{2 \left(\frac{1}{\sqrt{3}} \right)}{1 - \left(\frac{1}{\sqrt{3}} \right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} \\ &= \sqrt{3} \end{aligned}$$

Out of the given alternatives, only $\tan 60^\circ = \sqrt{3}$

Hence, (C) is correct.

Question 3:

If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$;

$0^\circ < A + B \leq 90^\circ$, $A > B$ find A and B.

Answer :

$$\tan(A + B) = \sqrt{3}$$

$$\Rightarrow \tan(A + B) = \tan 60$$

$$\Rightarrow A + B = 60 \dots (1)$$

$$\tan(A - B) = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan(A - B) = \tan 30$$

$$\Rightarrow A - B = 30 \dots (2)$$

On adding both equations, we obtain

$$2A = 90$$

$$\Rightarrow A = 45$$

From equation (1), we obtain

$$45 + B = 60$$

$$B = 15$$

Therefore, $\angle A = 45^\circ$ and $\angle B = 15^\circ$

Question 4:

State whether the following are true or false. Justify your answer.

- (i) $\sin (A + B) = \sin A + \sin B$
- (ii) The value of $\sin \theta$ increases as θ increases
- (iii) The value of $\cos \theta$ increases as θ increases
- (iv) $\sin \theta = \cos \theta$ for all values of θ
- (v) $\cot A$ is not defined for $A = 0^\circ$

Answer :

(i) $\sin (A + B) = \sin A + \sin B$

Let $A = 30^\circ$ and $B = 60^\circ$

$$\sin (A + B) = \sin (30^\circ + 60^\circ)$$

$$= \sin 90^\circ$$

$$= 1$$

$$\sin A + \sin B = \sin 30^\circ + \sin 60^\circ$$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1 + \sqrt{3}}{2}$$

Clearly, $\sin (A + B) \neq \sin A + \sin B$

Hence, the given statement is false.

(ii) The value of $\sin \theta$ increases as θ increases in the interval of $0^\circ < \theta < 90^\circ$ as

$$\sin 0^\circ = 0$$

$$\sin 30^\circ = \frac{1}{2} = 0.5$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}} = 0.707$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2} = 0.866$$

$$\sin 90^\circ = 1$$

Hence, the given statement is true.

$$(iii) \cos 0^\circ = 1$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2} = 0.866$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}} = 0.707$$

$$\cos 60^\circ = \frac{1}{2} = 0.5$$

$$\cos 90^\circ = 0$$

It can be observed that the value of $\cos \theta$ does not increase in the interval of $0^\circ < \theta < 90^\circ$.

Hence, the given statement is false.

$$(iv) \sin \theta = \cos \theta \text{ for all values of } \theta.$$

This is true when $\theta = 45^\circ$

$$\text{As } \sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}}$$

It is not true for all other values of θ .

$$\text{As } \sin 30^\circ = \frac{1}{2} \text{ and } \cos 30^\circ = \frac{\sqrt{3}}{2},$$

Hence, the given statement is false.

$$(v) \cot A \text{ is not defined for } A = 0^\circ$$

$$\text{As } \cot A = \frac{\cos A}{\sin A},$$

$$\cot 0^\circ = \frac{\cos 0^\circ}{\sin 0^\circ} = \frac{1}{0} = \text{undefined}$$

Hence, the given statement is true.