Newton's law of Gravitation: -

In 1660, Newton proposed a law of which states that, " Any two particles of matter anywhere in the universe attract each other with a force directly proportional to their masses and inversely proportional to square of distance between them". When the direction of the force being along the line joining the two particles.

Consider two bodies of mass m_1 and m_2 placed at a distance of r from each other then the force of attraction "f" between them is given as under: -

$$F \propto m_1 \times m_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(i)$$

$$F \propto 1/r^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(ii)$$

On comparing both the equations we have.

$$F \propto \frac{m_1 \times m_2}{r^2}$$

$$F = \frac{Gm_1m_2}{r^2}$$

Where "G" is the constant of proportionality and is known as universal gravitational constant. On S.I system its value has been calculated as $6.6734 \times 10^{-11} Nm^2/kg^2$.

Units of universal gravitational constant (G): -

The units of "G" depend upon the units of force, mass and distance. On S.I system where the units of force, mass and distance in Newton, kilogram and metre respectively, the units of "G" can be calculated as under: -

Since $$F = \frac{Gm_1m_2}{r^2}$$

$$\Rightarrow \quad G = \frac{Fr^2}{m_1m_2}$$

$$G = \frac{N \times m^2}{kg \times kg}$$

$$G = \frac{Nm^2}{kg^2}$$

From the above, it is clear that the units of universal gravitational constant "G" is $\frac{Nm^2}{kg^2}$

Gravitational force between the object of ordinary size: -

The value of G greatly is influenced by the size of the object. If the objects are small in size, then the gravitational force of attraction between them will also be small and hence no motion is observed in them. However, if at least one of the two bodies is large in size, then the gravitational force of attraction between them also becomes larger. In both the cases, the magnitude of force of attraction between them can be calculated by using the under given examples: -

$$F = \frac{Gm_1m_2}{r^2}$$

Where m_1 and m_2 is the mass of the two bodies and " r " as distance of separation between them and "G" is equivalent to the universal gravitational constant.

Newton's Third law of motion and gravitation: -

Newton's third law of motion states that "If a body exerts a force on another body, then the second body intern exerts an equal force and opposite force on the first body". This law holds good for gravitation also. Thus, when earth exerts a gravitational force of attraction on an opposite object, the object intern exerts an equal and opposite force on the earth. But it is the object only which, is pulled by the earth towards itself and no movement is observed in the earth. This can be explained as under: -

From Newton's second law of motion we have.

$$F = m \times a$$

Or $$a = F / m$$

From the above equation, it is clear that the acceleration produced in a body is inversely proportional to the mass of the body. Since the mass of the object is very small as compared to the large mass of the earth the acceleration produced will be greater in the object and least in the earth, which is not even observed and hence the object moves towards the earth.

Acceleration due to Gravity: -

When an object is falling from a height, its velocity increases uniformly, i-e, a uniform acceleration is produced in object by the gravitational pull of the earth. This uniform acceleration produced in a free falling body due to earth's gravitational pull is known as acceleration due to gravity. It is denoted by letter "g"

Consider a body of mass "m" is dropped from a distance of "R" from the centre of earth of mass "M". Then the gravitational force exerted by the earth is given by the relation.

$$F = \frac{GMm}{R^2} \quad \ldots\ldots\ldots\ldots\ldots\text{(i)}$$

Where G = Universal gravitational constant.

Since, the gravitational force produced acceleration in the falling body when.

$$F = m \times a \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

On comparing equation (i) & (ii) we have.

$$\cancel{m} \times a = \frac{GM\cancel{m}}{R^2}$$

$$a = \frac{Gm}{R^2}$$

Since, the acceleration produced by the earth is known as acceleration due to gravity, which is denoted by letter "g".

Thus,

$$G = \frac{Gm}{R^2}$$

From the above relation it is clear that the value of acceleration due to gravity does not depend upon the mass of the earth and the height of the object from the centre of the earth.

Units of acceleration due to gravity: -

On S.I system, mass is measured in kgs and the distance is measured in metres. Thus, the units of acceleration due to gravity can be calculated as under: -

$$g = \frac{Gm}{R^2}$$

$$\Rightarrow \quad g = \frac{Nm^2 \times kg}{kg^2 \times m \times m}$$

$$g = N/kg$$

But we know that

$$1N = 1kg \times 1m/sec^2$$

$$\therefore \quad g = \frac{kg \times m/sec^2}{kg}$$

$$\Rightarrow \quad g = \frac{kg \times m}{kg \times sec^2}$$

$$\Rightarrow \quad g = m/sec^2$$

$\therefore$ Acceleration due to gravity "g" has the same units of m/sec^2 as that of acceleration.

Mass of a body: -

It may be defined as the quantity of matter possessed by a body. It remains constant and does not change from place to place. it is denoted by letter "m" and is measured in kilograms. Infact, the mass of the body is the measure of its inertia and hence it is often termed as inertial mass and is measured in kilograms.

Measurement of mass with gravity: -

The gravitational pull of the earth attracts a body of a definite mass towards itself and is dependent on the mass of the object. Thus, this property can be utilised in the measurement of
masses of different bodies as described as under: -

Consider two bodies of mass "m_1" and "m_2" held at an equal distance from the centre of the earth. If "F_1" were the force of gravity with which the body of mass $m_1$is attached by the earth of mass Me, thus.

Newton's law of Gravitation: -

In 1660 Newton proposed a law which states that " Any two particles of matter anywhere in the universe attract each other". Thus,

For first body of mass m_1, we have

$$F_1 = \frac{Gme\, m_1}{R^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

Where, G is the universal gravitational constant. Similarly, if F_2 be the force of gravity with which the body of mass m_2 is attracted by the earth of mass Me. Thus,

For 2nd body of mass m_2 , we have

$$F_2 = \frac{Gme\, m_2}{R^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

Since, both the bodies are attracted by equal force. Then, $F_1= F_2$

Therefore, on comparing equation (i) & (ii) we have.

$$\frac{Gme\, m_1}{R^2} = \frac{Gme\, m_2}{R^2}$$

$$m_1 = m_2$$

From the above equation, it is clear that if two bodies are attracted by equal force then their masses are equal.

Weight of the body: -

The gravitational pull of the earth attracts every object towards its centre, which depends upon the mass and acceleration due to gravity of that body. Thus, the weight of a body maybe defined as the force with which, it is attracted towards the centre of the earth. Thus, force acting on a body

Force acting on a body = mass of the body x acceleration due to gravity

$$F = m \times g$$

But from the definition of the weight of a body,

$$W = F = m \times g$$

$$W = m \times g$$

Where "m" is the mass of the body "g" is acceleration due to gravity and "w" is the weight of the body.

Units of weight: - On S.I system, the unit of weight is same to that of the force i-e Newton which, is denoted by letter "N". It is called as the absolute unit of the weight. Apart from this, there is another unit called as the gravitational unit of the weight which, is kilogram wt, When 1kg wt. is the amount of the gravitational force which, acts on a body of mass 1kg.

Relation between Newton and Kilogram weight: -

Since, from the definition of the weight of a body, we have

$$W = m \times g$$

Thus, $1\text{kg wt} = 1\text{kg} \times 9.8\ \text{m/sec}^2$

$= 9.8\ \text{m/sec}^2$

But $1\text{kg} \times 1\text{m/sec}^2 = 1\text{Newton}$

$\therefore$ 1kg weight = 9.8 Newton's

Projectiles: -

A falling body having horizontal motion and vertical acceleration is called as projectile and the motion is referred to as projectile motion. The curved path which, a projectile acquires is called as Trajectory.

Characteristics of a projectile: - A projectile shows the following characteristics: -

1. It possesses a horizontal motion with a constant velocity.
2. It possesses vertical motion with a constant acceleration due to the force of gravity.
3. Its vertical acceleration is independent of horizontal motion and the vertical distance is $H = 1/2gt^2$.
4. The horizontal distance is travelled is dependent on the initial horizontal speed.

Escape velocity: -

The escape velocity of a body is defined as the minimum velocity with which, it must be projected so that it may escape away from the earth's gravitational pull into the outer space. It is mathematically expressed as under: -

$$V_e = \sqrt{2gr}$$

Where g = gravity of the earth = $9.8\ \text{m/sec}^2$

r = radius of the earth = 6.4×10^6m.

From the above expression, it is clear that the escape velocity does not depend on the mass of the object. For earth, its value is about 11.2km/sec. for mercury, it is 4.2km/sec and for Jupiter, the escape velocity is 61km/sec.

End........!