Chapter 9 Algebraic expressions

Exercise 9.1

Question 1:Identify the terms, their coefficients for each of the following expressions.

(i) $5xyz^2 - 3zy$(ii) $1 + x + x^2$(iii) $4x^2y^2 - 4x^2y^2z^2 + z^2$(iv) $3 - pq + qr - rp$

(v) $\frac{x}{2}+\frac{y}{2}-xy$

(vi) $0.3a - 0.6ab + 0.5b$

The terms and the respective coefficients of the given expressions are as follows.

-	Terms	Coefficients
(i)	$5xyz^2$	5
	$-3zy$	-3
(ii)	1	1
	x	1
	x^2	1
(iii)	$4x^2y^2$	4
	$-4x^2y^2z^2$	-4
	z^2	1
(iv)	3	3
	$-pq$	-1
	qr	1
	$-rp$	-1
(v)	$\frac{x}{2}$	$\frac{1}{2}$

	$\frac{y}{2}$ $-xy$	$\frac{1}{2}$ -1
(vi)	$0.3a$ $-0.6ab$ $0.5b$	0.3 -0.6 0.5

Question 2:Classify the following polynomials as monomials, binomials, trinomials. Which polynomials do not fit in any of these three categories?

$x + y$, 1000, $x + x^2 + x^3 + x^4$, $7 + y + 5x$, $2y - 3y^2$, $2y - 3y^2 + 4y^3$, $5x - 4y + 3xy$, $4z - 15z^2$, $ab + bc + cd + da$, pqr, $p^2q + pq^2$, $2p + 2q$

The given expressions are classified as

Monomials: 1000,pqr

Binomials: $x + y$, $2y - 3y^2$, $4z - 15z^2$, $p^2q + pq^2$, $2p + 2q$

Trinomials: $7 + y + 5x$, $2y - 3y^2 + 4y^3$, $5x - 4y + 3xy$

Polynomials that do not fit in any of these categories are

$x + x^2 + x^3 + x^4$, $ab + bc + cd + da$

Question 3:Add the following.

(i) $ab - bc$, $bc - ca$, $ca - ab$

(ii) $a - b + ab$, $b - c + bc$, $c - a + ac$

(iii) $2p^2q^2 - 3pq + 4$, $5 + 7pq - 3p^2q^2$

(iv) $l^2 + m^2$, $m^2 + n^2$, $n^2 + l^2$, $2lm + 2mn + 2nl$

The given expressions written in separate rows, with like terms one below the other and then the addition of these expressions are as follows.

(i)

$$\begin{array}{rrrr} & ab & -bc & \\ + & & bc & -ca \\ + & -ab & & +ca \\ \hline & & 0 & \end{array}$$

Thus, the sum of the given expressions is 0.

(ii)

$$\begin{array}{rrrrr} & a & -b & +ab & \\ + & & b & -c & +bc \\ + & -a & & c & +ac \\ \hline & & ab & +bc & +ac \end{array}$$

Thus, the sum of the given expressions is $ab + bc + ac$.

(iii)

$$\begin{array}{rrr} & 2p^2q^2 & -3pq+4 \\ + & -3p^2q^2 & +7pq+5 \\ \hline & -p^2q^2 & +4pq+9 \end{array}$$

Thus, the sum of the given expressions is $-p^2q^2 + 4pq + 9$.

(iv)

$$\begin{array}{rrrr} & l^2 & +m^2 & \\ + & & m^2 & +n^2 \\ + & l^2 & & +n^2 \\ + & & & 2lm+2mn+2nl \\ \hline & 2l^2 & +2m^2 & +2n^2+2lm+2mn+2nl \end{array}$$

Thus, the sum of the given expressions is $2(l^2 + m^2 + n^2 + lm + mn + nl)$.

Question 4:

(a) Subtract $4a - 7ab + 3b + 12$ from $12a - 9ab + 5b - 3$

(b) Subtract $3xy + 5yz - 7zx$ from $5xy - 2yz - 2zx + 10xyz$

(c) Subtract $4p^2q - 3pq + 5pq^2 - 8p + 7q - 10$ from $18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q$

The given expressions in separate rows, with like terms one below the other and then the subtraction of these expressions is as follows.

(a)

$$\begin{array}{r} 12a - 9ab + 5b - 3 \\ 4a - 7ab + 3b + 12 \\ (-) \quad (+) \quad (-) \quad (-) \\ \hline 8a - 2ab + 2b - 15 \end{array}$$

(b)

$$\begin{array}{r} 5xy - 2yz - 2zx + 10xyz \\ 3xy + 5yz - 7zx \\ (-) \quad (-) \quad (+) \\ \hline 2xy - 7yz + 5zx + 10xyz \end{array}$$

(c)

$$\begin{array}{r} 18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q \\ -10 - 8p + 7q - 3pq + 5pq^2 + 4p^2q \\ (+) \quad (+) \quad (-) \quad (+) \quad (-) \quad (-) \\ \hline 28 + 5p - 18q + 8pq - 7pq^2 + p^2q \end{array}$$

Exercise 9.2

Question 1:Find the product of the following pairs of monomials.

(i) 4, $7p$ (ii) $-4p$, $7p$ (iii) $-4p$, $7pq$

(iv) $4p^3$, $-3p$ (v) $4p$, 0

The product will be as follows.

(i) $4 \times 7p = 4 \times 7 \times p = 28p$

(ii) $-4p \times 7p = -4 \times p \times 7 \times p = (-4 \times 7) \times (p \times p) = -28\,p^2$

(iii) $-4p \times 7pq = -4 \times p \times 7 \times p \times q = (-4 \times 7) \times (p \times p \times q) = -28p^2q$

(iv) $4p^3 \times -3p = 4 \times (-3) \times p \times p \times p \times p = -12\,p^4$

(v) $4p \times 0 = 4 \times p \times 0 = 0$

Question 2:Find the areas of rectangles with the following pairs of monomials as their lengths and breadths respectively.

(p, q); ($10m$, $5n$); ($20x^2$, $5y^2$); ($4x$, $3x^2$); ($3mn$, $4np$)

We know that,

Area of rectangle = Length × Breadth

Area of 1st rectangle = $p \times q = pq$

Area of 2nd rectangle = $10m \times 5n = 10 \times 5 \times m \times n = 50\,mn$

Area of 3rd rectangle = $20x^2 \times 5y^2 = 20 \times 5 \times x^2 \times y^2 = 100\,x^2y^2$

Area of 4th rectangle = $4x \times 3x^2 = 4 \times 3 \times x \times x^2 = 12x^3$

Area of 5th rectangle = $3mn \times 4np = 3 \times 4 \times m \times n \times n \times p = 12mn^2p$

Question 3:Complete the table of products.

First monomial → / Second monomial ↓	$2x$	$-5y$	$3x^2$	$-4xy$	$7x^2y$	$-9x^2y^2$
$2x$	$4x^2$	…	…	…	…	…
$-5y$	…	…	$-15x^2y$	…	…	…
$3x^2$	…	…	…	…	…	…
$-4xy$	…	…	…	…	…	…
$7x^2y$	…	…	…	…	…	…
$-9x^2y^2$	…	…	…	…	…	…

The table can be completed as follows.

First monomial → / Second monomial ↓	$2x$	$-5y$	$3x^2$	$-4xy$	$7x^2y$	$-9x^2y^2$
$2x$	$4x^2$	$-10xy$	$6x^3$	$-8x^2y$	$14x^3y$	$-18x^3y^2$
$-5y$	$-10xy$	$25\ y^2$	$-15x^2y$	$20xy^2$	$-35x^2y^2$	$45x^2y^3$
$3x^2$	$6x^3$	$-15x^2y$	$9x^4$	$-12x^3y$	$21x^4y$	$-27x^4y^2$
$-4xy$	$-8x^2y$	$20xy^2$	$-12x^3y$	$16x^2y^2$	$-28x^3y^2$	$36x^3y^3$
$7x^2y$	$14x^3y$	$-35x^2y^2$	$21x^4y$	$-28x^3y^2$	$49x^4y^2$	$-63x^4y^3$
$-9x^2y^2$	$-18x^3y^2$	$45\ x^2y^3$	$-27x^4y^2$	$36x^3y^3$	$-63x^4y^3$	$81x^4y^4$

Question 4:Obtain the volume of rectangular boxes with the following length, breadth and height respectively.

(i) $5a$, $3a^2$, $7a^4$ (ii) $2p$, $4q$, $8r$ (iii) xy, $2x^2y$, $2xy^2$

(iv) a, $2b$, $3c$

Volume = Length × Breadth × Height

(i) Volume = $5a \times 3a^2 \times 7a^4 = 5 \times 3 \times 7 \times a \times a^2 \times a^4 = 105\, a^7$

(ii) Volume = $2p \times 4q \times 8r = 2 \times 4 \times 8 \times p \times q \times r = 64pqr$

(iii) Volume = $xy \times 2x^2y \times 2xy^2 = 2 \times 2 \times xy \times x^2y \times xy^2 = 4x^4y^4$

(iv) Volume = $a \times 2b \times 3\text{c} = 2 \times 3 \times a \times b \times \text{c} = 6ab\text{c}$

Exercise 9.3

Question 1:Carry out the multiplication of the expressions in each of the following pairs.

(i) $4p$, $q + r$ (ii) ab, $a - b$ (iii) $a + b$, $7a^2b^2$

(iv) $a^2 - 9$, $4a$ (v) $pq + qr + rp$, 0

(i) $(4p) \times (q + r) = (4p \times q) + (4p \times r) = 4pq + 4pr$

(ii) $(ab) \times (a - b) = (ab \times a) + [ab \times (-b)] = a^2b - ab^2$

(iii) $(a + b) \times (7a^2b^2) = (a \times 7a^2b^2) + (b \times 7a^2b^2) = 7a^3b^2 + 7a^2b^3$

(iv) $(a^2 - 9) \times (4a) = (a^2 \times 4a) + (-9) \times (4a) = 4a^3 - 36a$

(v) $(pq + qr + rp) \times 0 = (pq \times 0) + (qr \times 0) + (rp \times 0) = 0$

Question 2:Complete the table

---	First expression	Second Expression	Product
(i)	a	$b + c + d$	-
(ii)	$x + y - 5$	$5xy$	-
(iii)	p	$6p^2 - 7p + 5$	-
(iv)	$4p^2q^2$	$p^2 - q^2$	-
(v)	$a + b + c$	abc	-

The table can be completed as follows.

-	First expression	Second Expression	Product
(i)	a	$b + c + d$	$ab + ac + ad$
(ii)	$x + y - 5$	$5xy$	$5x^2y + 5xy^2 - 25xy$
(iii)	p	$6p^2 - 7p + 5$	$6p^3 - 7p^2 + 5p$
(iv)	$4p^2q^2$	$p^2 - q^2$	$4p^4q^2 - 4p^2q^4$
(v)	$a + b + c$	abc	$a^2bc + ab^2c + abc^2$

Question 3:Find the product.

(i) $(a^2) \times (2a^{22}) \times (4a^{26})$

(ii) $\left(\frac{2}{3}xy\right) \times \left(\frac{-9}{10}x^2y^2\right)$

(iii) $\left(-\frac{10}{3}pq^3\right) \times \left(\frac{6}{5}p^3q\right)$

(iv) $x \times x^2 \times x^3 \times x^4$

(i) $(a^2) \times (2a^{22}) \times (4a^{26}) = 2 \times 4 \times a^2 \times a^{22} \times a^{26} = 8a^{50}$

(ii) $\left(\frac{2}{3}xy\right)\times\left(\frac{-9}{10}x^2y^2\right)=\left(\frac{2}{3}\right)\times\left(\frac{-9}{10}\right)\times x\times y\times x^2\times y^2 = \frac{-3}{5}x^3y^3$

(iii) $\left(\frac{-10}{3}pq^3\right)\times\left(\frac{6}{5}p^3q\right)=\left(\frac{-10}{3}\right)\times\left(\frac{6}{5}\right)\times pq^3\times p^3q=-4p^4q^4$

(iv) $x \times x^2 \times x^3 \times x^4 = x^{10}$

Question 4:(a) Simplify 3x (4x −5) + 3 and find its values for (i) x = 3, (ii) $x=\frac{1}{2}$**.**

(b) *a* (a^2 + *a* + 1) + 5 and find its values for (i) *a* = 0, (ii) *a* = 1, (iii) *a* = −1.

(a) $3x\ (4x - 5) + 3 = 12x^2 - 15x + 3$

(i) For $x = 3$, $12x^2 - 15x + 3 = 12\ (3)^2 - 15(3) + 3$

$= 108 - 45 + 3$

$= 66$

(ii) For $x=\frac{1}{2}, 12x^2-15x+3=12\left(\frac{1}{2}\right)^2-15\left(\frac{1}{2}\right)+3$

$=12\times\frac{1}{4}-\frac{15}{2}+3$

$=3-\frac{15}{2}+3=6-\frac{15}{2}$

$=\frac{12-15}{2}=\frac{-3}{2}$

(b)$a\ (a^2 + a + 1) + 5 = a^3 + a^2 + a + 5$

(i) For$a = 0$, $a^3 + a^2 + a + 5 = 0 + 0 + 0 + 5 = 5$

(ii) For$a = 1$, $a^3 + a^2 + a + 5 = (1)^3 + (1)^2 + 1 + 5$

= 1 + 1 + 1 + 5 = 8

(iii) For $a = -1$, $a^3 + a^2 + a + 5 = (-1)^3 + (-1)^2 + (-1) + 5$

= − 1 + 1 − 1 + 5 = 4

Question 5:(a) Add: p (p − q), q (q− r) and r (r− p)

(b) Add: 2*x* (*z* − *x* − *y*) and 2*y* (*z* − *y* − *x*)

(c) Subtract: 3*l* (*l* − 4*m* + 5*n*) from 4*l* (10*n* − 3*m* + 2*l*)

(d) Subtract: 3*a* (*a* + *b* + *c*) − 2*b* (*a* − *b* + *c*) from 4*c* (− *a* + *b* + *c*)

(a) First expression = $p\,(p - q) = p^2 - pq$

Second expression = $q\,(q - r) = q^2 - qr$

Third expression = $r\,(r - p) = r^2 - pr$

Adding the three expressions, we obtain

$$\begin{array}{r} p^2 - pq \\ + \quad q^2 - qr \\ + \quad r^2 - pq \\ \hline p^2 - pq + q^2 - qr + r^2 - pq \end{array}$$

Therefore, the sum of the given expressions is $p^2 + q^2 + r^2 - pq - qr - rp$.

(b) First expression = $2x\,(z - x - y) = 2xz - 2x^2 - 2xy$

Second expression = $2y\,(z - y - x) = 2yz - 2y^2 - 2yx$

Adding the two expressions, we obtain

$$\begin{array}{r} 2xz - 2x^2 - 2xy \\ + \quad -2yx + 2yz - 2y^2 \\ \hline 2xz - 2x^2 - 4xy + 2yz - 2y^2 \end{array}$$

Therefore, the sum of the given expressions is $-2x^2 - 2y^2 - 4xy + 2yz + 2zx$.

(c) $3l\,(l - 4m + 5n) = 3l^2 - 12lm + 15ln$

$4l\ (10n - 3m + 2l) = 40ln - 12lm + 8l^2$

Subtracting these expressions, we obtain

$$\begin{array}{r} 40ln - 12lm + 8l^2 \\ 15ln - 12lm + 3l^2 \\ (-) \quad (+) \quad (-) \\ \hline +25ln \qquad +5l^2 \end{array}$$

Therefore, the result is $5l^2 + 25ln$.

(d) $3a\ (a + b + c) - 2b\ (a - b + c) = 3a^2 + 3ab + 3ac - 2ba + 2b^2 - 2bc$

$= 3a^2 + 2b^2 + ab + 3ac - 2bc$

$4c\ (-a + b + c) = -4ac + 4bc + 4c^2$

Subtracting these expressions, we obtain

$$\begin{array}{r} -4ac + 4bc + 4c^2 \\ 3ac - 2bc \qquad +3a^2 + 2b^2 + ab \\ (-) \quad (+) \qquad (-) \quad (-) \quad (-) \\ \hline -7ac + 6bc + 4c^2 - 3a^2 - 2b^2 - ab \end{array}$$

Therefore, the result is $-3a^2 - 2b^2 + 4c^2 - ab + 6bc - 7ac$.

Exercise 9.4

Question 1:Multiply the binomials.

(i) $(2x + 5)$ and $(4x - 3)$ (ii) $(y - 8)$ and $(3y - 4)$

(iii) $(2.5l - 0.5m)$ and $(2.5l + 0.5m)$ (iv) $(a + 3b)$ and $(x + 5)$

(v) $(2pq + 3q^2)$ and $(3pq - 2q^2)$

(vi) $\left(\frac{3}{4}a^2 + 3b^2\right)$ and $4\left(a^2 - \frac{2}{3}b^2\right)$

(i) $(2x + 5) \times (4x - 3) = 2x \times (4x - 3) + 5 \times (4x - 3)$

$= 8x^2 - 6x + 20x - 15$

$= 8x^2 + 14x - 15$ (By adding like terms)

(ii) $(y - 8) \times (3y - 4) = y \times (3y - 4) - 8 \times (3y - 4)$

$= 3y^2 - 4y - 24y + 32$

$= 3y^2 - 28y + 32$ (By adding like terms)

(iii) $(2.5l - 0.5m) \times (2.5l + 0.5m) = 2.5l \times (2.5l + 0.5m) - 0.5m\ (2.5l + 0.5m)$

$= 6.25l^2 + 1.25lm - 1.25lm - 0.25m^2$

$= 6.25l^2 - 0.25m^2$

(iv) $(a + 3b) \times (x + 5) = a \times (x + 5) + 3b \times (x + 5)$

$= ax + 5a + 3bx + 15b$

(v) $(2pq + 3q^2) \times (3pq - 2q^2) = 2pq \times (3pq - 2q^2) + 3q^2 \times (3pq - 2q^2)$

$= 6p^2q^2 - 4pq^3 + 9pq^3 - 6q^4$

$= 6p^2q^2 + 5pq^3 - 6q^4$

(vi) $\left(\frac{3}{4}a^2 + 3b^2\right) \times \left[4\left(a^2 - \frac{2}{3}b^2\right)\right] = \left(\frac{3}{4}a^2 + 3b^2\right) \times \left(4a^2 - \frac{8}{3}b^2\right)$

$= \frac{3}{4}a^2 \times \left(4a^2 - \frac{8}{3}b^2\right) + 3b^2 \times \left(4a^2 - \frac{8}{3}b^2\right)$

$= 3a^4 - 2a^2b^2 + 12b^2a^2 - 8b^4$

$= 3a^4 + 10a^2b^2 - 8b^4$

Question 2:Find the product.

(i) $(5 - 2x)\ (3 + x)$ (ii) $(x + 7y)\ (7x - y)$

(iii) $(a^2 + b)$ $(a + b^2)$ (iv) $(p^2 - q^2)$ $(2p + q)$

(i) $(5 - 2x)(3 + x) = 5(3 + x) - 2x(3 + x)$

$= 15 + 5x - 6x - 2x^2$

$= 15 - x - 2x^2$

(ii) $(x + 7y)(7x - y) = x(7x - y) + 7y(7x - y)$

$= 7x^2 - xy + 49xy - 7y^2$

$= 7x^2 + 48xy - 7y^2$

(iii) $(a^2 + b)(a + b^2) = a^2(a + b^2) + b(a + b^2)$

$= a^3 + a^2b^2 + ab + b^3$

(iv) $(p^2 - q^2)(2p + q) = p^2(2p + q) - q^2(2p + q)$

$= 2p^3 + p^2q - 2pq^2 - q^3$

Question 3:Simplify.

(i) $(x^2 - 5)(x + 5) + 25$ (ii) $(a^2 + 5)(b^3 + 3) + 5$

(iii) $(t + s^2)(t^2 - s)$ (iv) $(a + b)(c - d) + (a - b)(c + d) + 2(ac + bd)$

(v) $(x + y)(2x + y) + (x + 2y)(x - y)$(vi) $(x + y)(x^2 - xy + y^2)$

(vii) $(1.5x - 4y)(1.5x + 4y + 3) - 4.5x + 12y$(viii) $(a + b + c)(a + b - c)$

(i) $(x^2 - 5)(x + 5) + 25$

$= x^2(x + 5) - 5(x + 5) + 25$

$= x^3 + 5x^2 - 5x - 25 + 25$

$= x^3 + 5x^2 - 5x$

(ii) $(a^2 + 5)(b^3 + 3) + 5$

$= a^2 (b^3 + 3) + 5 (b^3 + 3) + 5$

$= a^2b^3 + 3a^2 + 5b^3 + 15 + 5$

$= a^2b^3 + 3a^2 + 5b^3 + 20$

(iii) $(t + s^2) (t^2 - s)$

$= t (t^2 - s) + s^2 (t^2 - s)$

$= t^3 - st + s^2t^2 - s^3$

(iv) $(a + b) (c - d) + (a - b) (c + d) + 2 (ac + bd)$

$= a (c - d) + b (c - d) + a (c + d) - b (c + d) + 2 (ac + bd)$

$= ac - ad + bc - bd + ac + ad - bc - bd + 2ac + 2bd$

$= (ac + ac + 2ac) + (ad - ad) + (bc - bc) + (2bd - bd - bd)$

$= 4ac$

(v) $(x + y) (2x + y) + (x + 2y) (x - y)$

$= x (2x + y) + y (2x + y) + x (x - y) + 2y (x - y)$

$= 2x^2 + xy + 2xy + y^2 + x^2 - xy + 2xy - 2y^2$

$= (2x^2 + x^2) + (y^2 - 2y^2) + (xy + 2xy - xy + 2xy)$

$= 3x^2 - y^2 + 4xy$

(vi) $(x + y) (x^2 - xy + y^2)$

$= x (x^2 - xy + y^2) + y (x^2 - xy + y^2)$

$= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3$

$= x^3 + y^3 + (xy^2 - xy^2) + (x^2y - x^2y)$

$= x^3 + y^3$

(vii) $(1.5x - 4y) (1.5x + 4y + 3) - 4.5x + 12y$

$= 1.5x (1.5x + 4y + 3) - 4y (1.5x + 4y + 3) - 4.5x + 12y$

$= 2.25x^2 + 6xy + 4.5x - 6xy - 16y^2 - 12y - 4.5x + 12y$

$= 2.25x^2 + (6xy - 6xy) + (4.5x - 4.5x) - 16y^2 + (12y - 12y)$

$= 2.25x^2 - 16y^2$

(viii) $(a + b + c)(a + b - c)$

$= a(a + b - c) + b(a + b - c) + c(a + b - c)$

$= a^2 + ab - ac + ab + b^2 - bc + ca + bc - c^2$

$= a^2 + b^2 - c^2 + (ab + ab) + (bc - bc) + (ca - ca)$

$= a^2 + b^2 - c^2 + 2ab$

Exercise 9.5

Question 1:Use a suitable identity to get each of the following products.

(i) $(x + 3)(x + 3)$ (ii) $(2y + 5)(2y + 5)$

(iii) $(2a - 7)(2a - 7)$ (iv) $\left(3a - \frac{1}{2}\right)\left(3a - \frac{1}{2}\right)$

(v) $(1.1m - 0.4)(1.1m + 0.4)$ (vi) $(a^2 + b^2)(-a^2 + b^2)$

(vii) $(6x - 7)(6x + 7)$ (viii) $(-a + c)(-a + c)$

(ix) $\left(\frac{x}{2} + \frac{3y}{4}\right)\left(\frac{x}{2} + \frac{3y}{4}\right)$ (x) $(7a - 9b)(7a - 9b)$

The products will be as follows.

(i) $(x + 3)(x + 3) = (x + 3)^2$

$= (x)^2 + 2(x)(3) + (3)^2$ $[(a + b)^2 = a^2 + 2ab + b^2]$

$= x^2 + 6x + 9$

(ii) $(2y + 5)(2y + 5) = (2y + 5)^2$

$= (2y)^2 + 2(2y)(5) + (5)^2$ $[(a + b)^2 = a^2 + 2ab + b^2]$

$= 4y^2 + 20y + 25$

(iii) $(2a - 7)(2a - 7) = (2a - 7)^2$

$= (2a)^2 - 2(2a)(7) + (7)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$

$= 4a^2 - 28a + 49$

(iv) $\left(3a-\frac{1}{2}\right)\left(3a-\frac{1}{2}\right)=\left(3a-\frac{1}{2}\right)^2$

$= (3a)^2-2(3a)\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$

$=9a^2-3a+\frac{1}{4}$

(v) $(1.1m - 0.4)(1.1m + 0.4)$

$= (1.1m)^2 - (0.4)^2$ $[(a + b)(a - b) = a^2 - b^2]$

$= 1.21m^2 - 0.16$

(vi) $(a^2 + b^2)(-a^2 + b^2) = (b^2 + a^2)(b^2 - a^2)$

$= (b^2)^2 - (a^2)^2$ $[(a + b)(a - b) = a^2 - b^2]$

$= b^4 - a^4$

(vii) $(6x - 7)(6x + 7) = (6x)^2 - (7)^2$ $[(a + b)(a - b) = a^2 - b^2]$

$= 36x^2 - 49$

(viii) $(-a + c)(-a + c) = (-a + c)^2$

$= (-a)^2 + 2(-a)(c) + (c)^2$ $[(a + b)^2 = a^2 + 2ab + b^2]$

$= a^2 - 2ac + c^2$

(ix) $\left(\frac{x}{2}+\frac{3y}{4}\right)\left(\frac{x}{2}+\frac{3y}{4}\right)=\left(\frac{x}{2}+\frac{3y}{4}\right)^2$

$=\left(\frac{x}{2}\right)^2+2\left(\frac{x}{2}\right)\left(\frac{3y}{4}\right)+\left(\frac{3y}{4}\right)^2$ $[(a + b)^2 = a^2 + 2ab + b^2]$

$=\frac{x^2}{4}+\frac{3xy}{4}+\frac{9y^2}{16}$

(x) $(7a - 9b)\ (7a - 9b) = (7a - 9b)^2$

$= (7a)^2 - 2(7a)(9b) + (9b)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$

$= 49a^2 - 126ab + 81b^2$

Question 2:Use the identity (x + a) (x + b) = x2 + (a + b)x + ab to find the following products.

(i) $(x + 3)\ (x + 7)$ (ii) $(4x + 5)\ (4x + 1)$

(iii) $(4x - 5)\ (4x - 1)$ (iv) $(4x + 5)\ (4x - 1)$

(v) $(2x + 5y)\ (2x + 3y)$ (vi) $(2a^2 + 9)\ (2a^2 + 5)$

(vii) $(xyz - 4)\ (xyz - 2)$

The products will be as follows.

(i) $(x + 3)\ (x + 7) = x^2 + (3 + 7)\ x + (3)\ (7)$

$= x^2 + 10x + 21$

(ii) $(4x + 5)\ (4x + 1) = (4x)^2 + (5 + 1)\ (4x) + (5)\ (1)$

$= 16x^2 + 24x + 5$

(iii) $(4x-5)(4x-1)=(4x)^2+\left[(-5)+(-1)\right](4x)+(-5)(-1)$

$= 16x^2 - 24x + 5$

(iv) $(4x+5)(4x-1)=(4x)^2+\left[(5)+(-1)\right](4x)+(5)(-1)$

$= 16x^2 + 16x - 5$

(v) $(2x + 5y)(2x + 3y) = (2x)^2 + (5y + 3y)(2x) + (5y)(3y) = 4x^2 + 16xy + 15y^2$

(vi) $(2a^2 + 9)(2a^2 + 5) = (2a^2)^2 + (9 + 5)(2a^2) + (9)(5) = 4a^4 + 28a^2 + 45$

(vii) $(xyz - 4)(xyz - 2)$

$= (xyz)^2 + [(-4) + (-2)](xyz) + (-4)(-2)$

$= x^2y^2z^2 - 6xyz + 8$

Question 3:Find the following squares by suing the identities.

(i) $(b - 7)^2$ (ii) $(xy + 3z)^2$ (iii) $(6x^2 - 5y)^2$

(iv) $\left(\frac{2}{3}m + \frac{3}{2}n\right)^2$ (v) $(0.4p - 0.5q)^2$ (vi) $(2xy + 5y)^2$

(i) $(b - 7)^2 = (b)^2 - 2(b)(7) + (7)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$

$= b^2 - 14b + 49$

(ii) $(xy + 3z)^2 = (xy)^2 + 2(xy)(3z) + (3z)^2$ $[(a + b)^2 = a^2 + 2ab + b^2]$

$= x^2y^2 + 6xyz + 9z^2$

(iii) $(6x^2 - 5y)^2 = (6x^2)^2 - 2(6x^2)(5y) + (5y)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$

$= 36x^4 - 60x^2y + 25y^2$

(iv) $\left(\frac{2}{3}m + \frac{3}{2}n\right)^2 = \left(\frac{2}{3}m\right)^2 + 2\left(\frac{2}{3}m\right)\left(\frac{3}{2}n\right) + \left(\frac{3}{2}n\right)^2$ $[(a + b)^2 = a^2 + 2ab + b^2]$

$= \frac{4}{9}m^2 + 2mn + \frac{9}{4}n^2$

(v) $(0.4p - 0.5q)^2 = (0.4p)^2 - 2(0.4p)(0.5q) + (0.5q)^2$

$[(a - b)^2 = a^2 - 2ab + b^2]$

$= 0.16p^2 - 0.4pq + 0.25q^2$

(vi) $(2xy + 5y)^2 = (2xy)^2 + 2(2xy)(5y) + (5y)^2$

$[(a + b)^2 = a^2 + 2ab + b^2]$

$= 4x^2y^2 + 20xy^2 + 25y^2$

Question 4:Simplify.

(i) $(a^2 - b^2)^2$ (ii) $(2x + 5)^2 - (2x - 5)^2$

(iii) $(7m - 8n)^2 + (7m + 8n)^2$ (iv) $(4m + 5n)^2 + (5m + 4n)^2$

(v) $(2.5p - 1.5q)^2 - (1.5p - 2.5q)^2$

(vi) $(ab + bc)^2 - 2ab^2c$ (vii) $(m^2 - n^2m)^2 + 2m^3n^2$

(i) $(a^2 - b^2)^2 = (a^2)^2 - 2(a^2)(b^2) + (b^2)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$

$= a^4 - 2a^2b^2 + b^4$

(ii) $(2x + 5)^2 - (2x - 5)^2 = (2x)^2 + 2(2x)(5) + (5)^2 - [(2x)^2 - 2(2x)(5) + (5)^2]$

$[(a - b)^2 = a^2 - 2ab + b^2]$

$[(a + b)^2 = a^2 + 2ab + b^2]$

$= 4x^2 + 20x + 25 - [4x^2 - 20x + 25]$

$= 4x^2 + 20x + 25 - 4x^2 + 20x - 25 = 40x$

(iii) $(7m - 8n)^2 + (7m + 8n)^2$

$= (7m)^2 - 2(7m)(8n) + (8n)^2 + (7m)^2 + 2(7m)(8n) + (8n)^2$

$[(a - b)^2 = a^2 - 2ab + b^2$and $(a + b)^2 = a^2 + 2ab + b^2]$

$= 49m^2 - 112mn + 64n^2 + 49m^2 + 112mn + 64n^2$

$= 98m^2 + 128n^2$

(iv) $(4m + 5n)^2 + (5m + 4n)^2$

$= (4m)^2 + 2(4m)(5n) + (5n)^2 + (5m)^2 + 2(5m)(4n) + (4n)^2$

[$(a + b)^2 = a^2 + 2ab + b^2$]

$= 16m^2 + 40mn + 25n^2 + 25m^2 + 40mn + 16n^2$

$= 41m^2 + 80mn + 41n^2$

(v) $(2.5p - 1.5q)^2 - (1.5p - 2.5q)^2$

$= (2.5p)^2 - 2(2.5p)\,(1.5q) + (1.5q)^2 - [(1.5p)^2 - 2(1.5p)(2.5q) + (2.5q)^2]$

$[(a - b)^2 = a^2 - 2ab + b^2\ ]$

$= 6.25p^2 - 7.5pq + 2.25q^2 - [2.25p^2 - 7.5pq + 6.25q^2]$

$= 6.25p^2 - 7.5pq + 2.25q^2 - 2.25p^2 + 7.5pq - 6.25q^2]$

$= 4p^2 - 4q^2$

(vi) $(ab + bc)^2 - 2ab^2c$

$= (ab)^2 + 2(ab)(bc) + (bc)^2 - 2ab^2c$ $[(a + b)^2 = a^2 + 2ab + b^2\ ]$

$= a^2b^2 + 2ab^2c + b^2c^2 - 2ab^2c$

$= a^2b^2 + b^2c^2$

(vii) $(m^2 - n^2m)^2 + 2m^3n^2$

$= (m^2)^2 - 2(m^2)\,(n^2m) + (n^2m)^2 + 2m^3n^2$ $[(a - b)^2 = a^2 - 2ab + b^2\ ]$

$= m^4 - 2m^3n^2 + n^4m^2 + 2m^3n^2$

$= m^4 + n^4m^2$

Question 5:Show that

(i) $(3x + 7)^2 - 84x = (3x - 7)^2$ (ii) $(9p - 5q)^2 + 180pq = (9p + 5q)^2$

(iii) $\left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn = \frac{16}{9}m^2 + \frac{9}{16}n^2$

(iv) $(4pq + 3q)^2 - (4pq - 3q)^2 = 48pq^2$

(v) $(a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a) = 0$

(i) L.H.S = $(3x + 7)^2 - 84x$

$= (3x)^2 + 2(3x)(7) + (7)^2 - 84x$

$= 9x^2 + 42x + 49 - 84x$

$= 9x^2 - 42x + 49$

R.H.S = $(3x - 7)^2 = (3x)^2 - 2(3x)(7) + (7)^2$

$= 9x^2 - 42x + 49$

L.H.S = R.H.S

(ii) L.H.S = $(9p - 5q)^2 + 180pq$

$= (9p)^2 - 2(9p)(5q) + (5q)^2 - 180pq$

$= 81p^2 - 90pq + 25q^2 + 180pq$

$= 81p^2 + 90pq + 25q^2$

R.H.S = $(9p + 5q)^2$

$= (9p)^2 + 2(9p)(5q) + (5q)^2$

$= 81p^2 + 90pq + 25q^2$

L.H.S = R.H.S

(iii) L.H.S = $\left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn$

$$= \left(\frac{4}{3}m\right)^2 - 2\left(\frac{4}{3}m\right)\left(\frac{3}{4}n\right) + \left(\frac{3}{4}n\right)^2 + 2mn$$
$$= \frac{16}{9}m^2 - 2mn + \frac{9}{16}n^2 + 2mn$$
$$= \frac{16}{9}m^2 + \frac{9}{16}n^2 = \text{R.H.S.}$$

(iv) L.H.S = $(4pq + 3q)^2 - (4pq - 3q)^2$

$= (4pq)^2 + 2(4pq)(3q) + (3q)^2 - [(4pq)^2 - 2(4pq)\,(3q) + (3q)^2]$

$= 16p^2q^2 + 24pq^2 + 9q^2 - [16p^2q^2 - 24pq^2 + 9q^2]$

$= 16p^2q^2 + 24pq^2 + 9q^2 - 16p^2q^2 + 24pq^2 - 9q^2$

$= 48pq^2$ = R.H.S

(v) L.H.S = $(a - b)\,(a + b) + (b - c)\,(b + c) + (c - a)\,(c + a)$

$= (a^2 - b^2) + (b^2 - c^2) + (c^2 - a^2) = 0$ = R.H.S.

Question 5:Show that

(i) $(3x + 7)^2 - 84x = (3x - 7)^2$ (ii) $(9p - 5q)^2 + 180pq = (9p + 5q)^2$

(iii) $\left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn = \frac{16}{9}m^2 + \frac{9}{16}n^2$

(iv) $(4pq + 3q)^2 - (4pq - 3q)^2 = 48pq^2$

(v) $(a - b)\,(a + b) + (b - c)\,(b + c) + (c - a)\,(c + a) = 0$

(i) L.H.S = $(3x + 7)^2 - 84x$

$= (3x)^2 + 2(3x)(7) + (7)^2 - 84x$

$= 9x^2 + 42x + 49 - 84x$

$= 9x^2 - 42x + 49$

R.H.S = $(3x - 7)^2 = (3x)^2 - 2(3x)(7) + (7)^2$

$= 9x^2 - 42x + 49$

L.H.S = R.H.S

(ii) L.H.S = $(9p - 5q)^2 + 180pq$

$= (9p)^2 - 2(9p)(5q) + (5q)^2 - 180pq$

$= 81p^2 - 90pq + 25q^2 + 180pq$

$= 81p^2 + 90pq + 25q^2$

R.H.S = $(9p + 5q)^2$

= $(9p)^2 + 2(9p)(5q) + (5q)^2$

= $81p^2 + 90pq + 25q^2$

L.H.S = R.H.S

(iii) L.H.S = $\left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn$

$$= \left(\frac{4}{3}m\right)^2 - 2\left(\frac{4}{3}m\right)\left(\frac{3}{4}n\right) + \left(\frac{3}{4}n\right)^2 + 2mn$$
$$= \frac{16}{9}m^2 - 2mn + \frac{9}{16}n^2 + 2mn$$
$$= \frac{16}{9}m^2 + \frac{9}{16}n^2 = \text{R.H.S.}$$

(iv) L.H.S = $(4pq + 3q)^2 - (4pq - 3q)^2$

= $(4pq)^2 + 2(4pq)(3q) + (3q)^2 - [(4pq)^2 - 2(4pq)(3q) + (3q)^2]$

= $16p^2q^2 + 24pq^2 + 9q^2 - [16p^2q^2 - 24pq^2 + 9q^2]$

= $16p^2q^2 + 24pq^2 + 9q^2 - 16p^2q^2 + 24pq^2 - 9q^2$

= $48pq^2$ = R.H.S

(v) L.H.S = $(a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a)$

= $(a^2 - b^2) + (b^2 - c^2) + (c^2 - a^2) = 0$ = R.H.S.

Question 6:Using identities, evaluate.

(i) 71^2 (ii) 99^2 (iii) 102^2(iv) 998^2

(v) $(5.2)^2$ (vi) 297 × 303 (vii) 78 × 82

(viii) 8.9^2 (ix) 1.05 × 9.5

(i) $71^2 = (70 + 1)^2$

$= (70)^2 + 2(70)\,(1) + (1)^2$ $[(a + b)^2 = a^2 + 2ab + b^2]$

$= 4900 + 140 + 1 = 5041$

(ii) $99^2 = (100 - 1)^2$

$= (100)^2 - 2(100)\,(1) + (1)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$

$= 10000 - 200 + 1 = 9801$

(iii) $102^2 = (100 + 2)^2$

$= (100)^2 + 2(100)(2) + (2)^2$ $[(a + b)^2 = a^2 + 2ab + b^2]$

$= 10000 + 400 + 4 = 10404$

(iv) $998^2 = (1000 - 2)^2$

$= (1000)^2 - 2(1000)(2) + (2)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$

$= 1000000 - 4000 + 4 = 996004$

(v) $(5.2)^2 = (5.0 + 0.2)^2$

$= (5.0)^2 + 2(5.0)\,(0.2) + (0.2)^2$ $[(a + b)^2 = a^2 + 2ab + b^2]$

$= 25 + 2 + 0.04 = 27.04$

(vi) $297 \times 303 = (300 - 3) \times (300 + 3)$

$= (300)^2 - (3)^2$ $[(a + b)\,(a - b) = a^2 - b^2]$

$= 90000 - 9 = 89991$

(vii) $78 \times 82 = (80 - 2)\,(80 + 2)$

$= (80)^2 - (2)^2$ $[(a + b)\,(a - b) = a^2 - b^2]$

$= 6400 - 4 = 6396$

(viii) $8.9^2 = (9.0 - 0.1)^2$

$= (9.0)^2 - 2(9.0)\,(0.1) + (0.1)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$

$= 81 - 1.8 + 0.01 = 79.21$

(ix) $1.05 \times 9.5 = 1.05 \times 0.95 \times 10$

$= (1 + 0.05)\ (1 - 0.05) \times 10$

$= [(1)^2 - (0.05)^2] \times 10$

$= [1 - 0.0025] \times 10$ $[(a + b)\ (a - b) = a^2 - b^2]$

$= 0.9975 \times 10 = 9.975$

Question 7:Using a2 − b2 = (a + b) (a − b), find

(i) $51^2 - 49^2$ (ii) $(1.02)^2 - (0.98)^2$ (iii) $153^2 - 147^2$

(iv) $12.1^2 - 7.9^2$

(i) $51^2 - 49^2 = (51 + 49)\ (51 - 49)$

$= (100)\ (2) = 200$

(ii) $(1.02)^2 - (0.98)^2 = (1.02 + 0.98)\ (1.02 - 0.98)$

$= (2)\ (0.04) = 0.08$

(iii) $153^2 - 147^2 = (153 + 147)\ (153 - 147)$

$= (300)\ (6) = 1800$

(iv) $12.1^2 - 7.9^2 = (12.1 + 7.9)\ (12.1 - 7.9)$

$= (20.0)\ (4.2) = 84$

Question 8:Using (x + a) (x + b) = x2 + (a + b) x + ab, find

(i) 103×104 (ii) 5.1×5.2 (iii) 103×98 (iv) 9.7×9.8

(i) $103 \times 104 = (100 + 3)\ (100 + 4)$

$= (100)^2 + (3 + 4)\ (100) + (3)\ (4)$

$= 10000 + 700 + 12 = 10712$

(ii) $5.1 \times 5.2 = (5 + 0.1)\ (5 + 0.2)$

$= (5)^2 + (0.1 + 0.2)\ (5) + (0.1)\ (0.2)$

$= 25 + 1.5 + 0.02 = 26.52$

(iii) $103 \times 98 = (100 + 3)(100 - 2)$

$= (100)^2 + [3 + (-2)](100) + (3)(-2)$

$= 10000 + 100 - 6$

$= 10094$

(iv) $9.7 \times 9.8 = (10 - 0.3)(10 - 0.2)$

$= (10)^2 + [(-0.3) + (-0.2)](10) + (-0.3)(-0.2)$

$= 100 + (-0.5)10 + 0.06 = 100.06 - 5 = 95.06$

What have we Discussed

1. Expressions are formed from **variables** and **constants.**
2. Terms are aided to form **expressions.** Terms themselves are formed as product of • **factors.**

   Expressions that contain exactly one, two and three terms are called *monomials, binomials* and *trinomials* respectively. In general, any expression containing one or more terms with non-zero coefficients (and with variables having non- negative exponents) is called a **polynomial.**
4. **Like** terms are formed from the same Variables and the powers of these variables are the same, too. Coefficients of like terms need not be the same.
5. While adding (or subtracting) polynomials, first look for like terms and add (or subtract) them; then handle the unlike terms.
6. There are number of situations in which we need to multiply algebraic expressions: for example, in finding area of a rectangle, the sides of which are given as expressions.
7. A monomial multiplied by a monomial always gives a monomial.
8. While multiplying a polynomial by a monomial, we multiply every term in the polynomial by the monomial.
9. In canying out the multiplication of a polynomial by a binomial (or trinomial), we multiply term by term, i.e., every term of the polynomial is multiplied by every term in the binomial (or trinomial).

Note that in such multiplication, we may get terms in the product which are like and have to be combined.

10. An **identity** is an equality, which is true for all values of the variables in the equality.
On the other hand, an equation is true only for certain values of its variables. An equation is not an identity.

11. The following are the standard identities:

$(a + b)^2 = a^2 + 2ab + b^2$ (I)

$(a - b)^2 = a^2 - lab + b^2$ *(II)*

$(a + b)(a - b) = a^2 - b^2$ *(III)*

12. Another useful identity is $(x + a)(x + b) = x^2 + (a + b)x + ab$ (IV)

13. The above four identities are useful in carrying out squares and products of algebraic expressions.
They also allow easy alternative methods to calculate products of numbers and so on.