Factorization

Exercise 14.1

Question 1:Find the common factors of the terms

(i) $12x$, 36

(ii) $2y$, $22xy$

(iii) $14pq$, $28p^2q^2$

(iv) $2x$, $3x^2$, 4

(v) $6abc$, $24ab^2$, $12a^2b$

(vi) $16x^3$, $-4x^2$, $32x$

(vii) $10pq$, $20qr$, $30rp$

(viii) $3x^2y^3$, $10x^3y^2$, $6x^2y^2z$

(i) $12x = 2 \times 2 \times 3 \times x$

$36 = 2 \times 2 \times 3 \times 3$

The common factors are 2, 2, 3.

And, $2 \times 2 \times 3 = 12$

(ii) $2y = 2 \times y$

$22xy = 2 \times 11 \times x \times y$

The common factors are 2, y.

And, $2 \times y = 2y$

(iii) $14pq = 2 \times 7 \times p \times q$

$28p^2q^2 = 2 \times 2 \times 7 \times p \times p \times q \times q$

The common factors are 2, 7, p, q.

And, $2 \times 7 \times p \times q = 14pq$

(iv) $2x = 2 \times x$

$3x^2 = 3 \times x \times x$

$4 = 2 \times 2$

The common factor is 1.

(v) $6abc = 2 \times 3 \times a \times b \times c$

$24ab^2 = 2 \times 2 \times 2 \times 3 \times a \times b \times b$

$12a^2b = 2 \times 2 \times 3 \times a \times a \times b$

The common factors are 2, 3, a, b.

And, $2 \times 3 \times a \times b = 6ab$

(vi) $16x^3 = 2 \times 2 \times 2 \times 2 \times x \times x \times x$

$-4x^2 = -1 \times 2 \times 2 \times x \times x$

$32x = 2 \times 2 \times 2 \times 2 \times 2 \times x$

The common factors are 2, 2, x.

And, $2 \times 2 \times x = 4x$

(vii) $10pq = 2 \times 5 \times p \times q$

$20qr = 2 \times 2 \times 5 \times q \times r$

$30rp = 2 \times 3 \times 5 \times r \times p$

The common factors are 2, 5.

And, $2 \times 5 = 10$

(viii) $3x^2y^3 = 3 \times x \times x \times y \times y \times y$

$10x^3y^2 = 2 \times 5 \times x \times x \times x \times y \times y$

$6x^2y^2z = 2 \times 3 \times x \times x \times y \times y \times z$

The common factors are x, x, y, y.

And,

$x \times x \times y \times y = x^2y^2$

Question 2:Factorise the following expressions

(i) $7x - 42$

(ii) $6p - 12q$

(iii) $7a^2 + 14a$

(iv) $-16z + 20z^3$

(v) $20l^2m + 30\ alm$

(vi) $5x^2y - 15xy^2$

(vii) $10a^2 - 15b^2 + 20c^2$

(viii) $-4a^2 + 4ab - 4\ ca$

(ix) $x^2yz + xy^2z + xyz^2$

(x) $ax^2y + bxy^2 + cxyz$

(i) $7x = 7 \times x$

$42 = 2 \times 3 \times 7$

The common factor is 7.

$\therefore 7x - 42 = (7 \times x) - (2 \times 3 \times 7) = 7(x - 6)$

(ii) $6p = 2 \times 3 \times p$

$12q = 2 \times 2 \times 3 \times q$

The common factors are 2 and 3.

$\therefore 6p - 12q = (2 \times 3 \times p) - (2 \times 2 \times 3 \times q)$

$= 2 \times 3\ [p - (2 \times q)]$

$= 6\ (p - 2q)$

(iii) $7a^2 = 7 \times a \times a$

$14a = 2 \times 7 \times a$

The common factors are 7 and a.

$\therefore 7a^2 + 14a = (7 \times a \times a) + (2 \times 7 \times a)$

$= 7 \times a\ [a + 2] = 7a\ (a + 2)$

(iv) $16z = 2 \times 2 \times 2 \times 2 \times z$

$20z^3 = 2 \times 2 \times 5 \times z \times z \times z$

The common factors are 2, 2, and z.

$\therefore -16z + 20z^3 = -(2 \times 2 \times 2 \times 2 \times z) + (2 \times 2 \times 5 \times z \times z \times z)$

$= (2 \times 2 \times z)[-(2 \times 2) + (5 \times z \times z)]$

$= 4z(-4 + 5z^2)$

(v) $20l^2m = 2 \times 2 \times 5 \times l \times l \times m$

$30alm = 2 \times 3 \times 5 \times a \times l \times m$

The common factors are 2, 5, l, and m.

$\therefore 20l^2m + 30alm = (2 \times 2 \times 5 \times l \times l \times m) + (2 \times 3 \times 5 \times a \times l \times m)$

$= (2 \times 5 \times l \times m)[(2 \times l) + (3 \times a)]$

$= 10lm(2l + 3a)$

(vi) $5x^2y = 5 \times x \times x \times y$

$15xy^2 = 3 \times 5 \times x \times y \times y$

The common factors are 5, x, and y.

$\therefore 5x^2y - 15xy^2 = (5 \times x \times x \times y) - (3 \times 5 \times x \times y \times y)$

$= 5 \times x \times y[x - (3 \times y)]$

$= 5xy(x - 3y)$

(vii) $10a^2 = 2 \times 5 \times a \times a$

$15b^2 = 3 \times 5 \times b \times b$

$20c^2 = 2 \times 2 \times 5 \times c \times c$

The common factor is 5.

$10a^2 - 15b^2 + 20c^2 = (2 \times 5 \times a \times a) - (3 \times 5 \times b \times b) + (2 \times 2 \times 5 \times c \times c)$

$= 5[(2 \times a \times a) - (3 \times b \times b) + (2 \times 2 \times c \times c)]$

$= 5(2a^2 - 3b^2 + 4c^2)$

(viii) $4a^2 = 2 \times 2 \times a \times a$

$4ab = 2 \times 2 \times a \times b$

$4ca = 2 \times 2 \times c \times a$

The common factors are 2, 2, and a.

$\therefore -4a^2 + 4ab - 4ca = -(2 \times 2 \times a \times a) + (2 \times 2 \times a \times b) - (2 \times 2 \times c \times a)$

$= 2 \times 2 \times a\,[-(a) + b - c]$

$= 4a\,(-a + b - c)$

(ix) $x^2yz = x \times x \times y \times z$

$xy^2z = x \times y \times y \times z$

$xyz^2 = x \times y \times z \times z$

The common factors are x, y, and z.

$\therefore x^2yz + xy^2z + xyz^2 = (x \times x \times y \times z) + (x \times y \times y \times z) + (x \times y \times z \times z)$

$= x \times y \times z\,[x + y + z]$

$= xyz\,(x + y + z)$

(x) $ax^2y = a \times x \times x \times y$

$bxy^2 = b \times x \times y \times y$

$cxyz = c \times x \times y \times z$

The common factors are x and y.

$ax^2y + bxy^2 + cxyz = (a \times x \times x \times y) + (b \times x \times y \times y) + (c \times x \times y \times z)$

$= (x \times y)\,[(a \times x) + (b \times y) + (c \times z)]$

$= xy\,(ax + by + cz)$

Question 3:Factorise

(i) $x^2 + xy + 8x + 8y$

(ii) $15xy - 6x + 5y - 2$

(iii) $ax + bx - ay - by$

(iv) $15pq + 15 + 9q + 25p$

(v) $z - 7 + 7xy - xyz$

(i) $x^2 + xy + 8x + 8y = x \times x + x \times y + 8 \times x + 8 \times y$

$= x\,(x + y) + 8\,(x + y)$

$= (x + y)\,(x + 8)$

(ii) $15xy - 6x + 5y - 2 = 3 \times 5 \times x \times y - 3 \times 2 \times x + 5 \times y - 2$

$= 3x\,(5y - 2) + 1\,(5y - 2)$

$= (5y - 2)\,(3x + 1)$

(iii) $ax + bx - ay - by = a \times x + b \times x - a \times y - b \times y$

$= x\,(a + b) - y\,(a + b)$

$= (a + b)\,(x - y)$

(iv) $15pq + 15 + 9q + 25p = 15pq + 9q + 25p + 15$

$= 3 \times 5 \times p \times q + 3 \times 3 \times q + 5 \times 5 \times p + 3 \times 5$

$= 3q\,(5p + 3) + 5\,(5p + 3)$

$= (5p + 3)\,(3q + 5)$

(v) $z - 7 + 7xy - xyz = z - x \times y \times z - 7 + 7 \times x \times y$

$= z\,(1 - xy) - 7\,(1 - xy)$

$= (1 - xy)\,(z - 7)$

Exercise 14.2

Question 1:Factorise the following expressions.

(i) $a^2 + 8a + 16$

(ii) $p^2 - 10p + 25$

(iii) $25m^2 + 30m + 9$

(iv) $49y^2 + 84yz + 36z^2$

(v) $4x^2 - 8x + 4$

(vi) $121b^2 - 88bc + 16c^2$

(vii) $(l + m)^2 - 4lm$ (**Hint**: Expand $(l + m)^2$ first)

(viii) $a^4 + 2a^2b^2 + b^4$

(i) $a^2 + 8a + 16 = (a)^2 + 2 \times a \times 4 + (4)^2$

$= (a + 4)^2$ $[(x + y)^2 = x^2 + 2xy + y^2]$

(ii) $p^2 - 10p + 25 = (p)^2 - 2 \times p \times 5 + (5)^2$

$= (p - 5)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$

(iii) $25m^2 + 30m + 9 = (5m)^2 + 2 \times 5m \times 3 + (3)^2$

$= (5m + 3)^2$ $[(a + b)^2 = a^2 + 2ab + b^2]$

(iv) $49y^2 + 84yz + 36z^2 = (7y)^2 + 2 \times (7y) \times (6z) + (6z)^2$

$= (7y + 6z)^2$ $[(a + b)^2 = a^2 + 2ab + b^2]$

(v) $4x^2 - 8x + 4 = (2x)^2 - 2\ (2x)\ (2) + (2)^2$

$= (2x - 2)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$

$= [(2)\ (x - 1)]^2 = 4(x - 1)^2$

(vi) $121b^2 - 88bc + 16c^2 = (11b)^2 - 2\ (11b)\ (4c) + (4c)^2$

$= (11b - 4c)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$

(vii) $(l + m)^2 - 4lm = l^2 + 2lm + m^2 - 4lm$

$= l^2 - 2lm + m^2$

$= (l - m)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$

(viii) $a^4 + 2a^2b^2 + b^4 = (a^2)^2 + 2\ (a^2)\ (b^2) + (b^2)^2$

$= (a^2 + b^2)^2$ $[(a + b)^2 = a^2 + 2ab + b^2]$

Question 2:Factorise

(i) $4p^2 - 9q^2$

(ii) $63a^2 - 112b^2$

(iii) $49x^2 - 36$

(iv) $16x^5 - 144x^3$

(v) $(l + m)^2 - (l - m)^2$

(vi) $9x^2y^2 - 16$

(vii) $(x^2 - 2xy + y^2) - z^2$

(viii) $25a^2 - 4b^2 + 28bc - 49c^2$

(i) $4p^2 - 9q^2 = (2p)^2 - (3q)^2$

$= (2p + 3q)\ (2p - 3q)\ [a^2 - b^2 = (a - b)\ (a + b)]$

(ii) $63a^2 - 112b^2 = 7(9a^2 - 16b^2)$

$= 7[(3a)^2 - (4b)^2]$

$= 7(3a + 4b)\ (3a - 4b)\ [a^2 - b^2 = (a - b)\ (a + b)]$

(iii) $49x^2 - 36 = (7x)^2 - (6)^2$

$= (7x - 6)\ (7x + 6)\ [a^2 - b^2 = (a - b)\ (a + b)]$

(iv) $16x^5 - 144x^3 = 16x^3(x^2 - 9)$

$= 16\ x^3\ [(x)^2 - (3)^2]$

$= 16\ x^3(x - 3)\ (x + 3)\ [a^2 - b^2 = (a - b)\ (a + b)]$

(v) $(l + m)^2 - (l - m)^2 = [(l + m) - (l - m)]\ [(l + m) + (l - m)]$

[Using identity $a^2 - b^2 = (a - b)\ (a + b)$]

$= (l + m - l + m)\ (l + m + l - m)$

$= 2m \times 2l$

$= 4ml$

$= 4lm$

(vi) $9x^2y^2 - 16 = (3xy)^2 - (4)^2$

$= (3xy - 4)\ (3xy + 4)\ [a^2 - b^2 = (a - b)\ (a + b)]$

(vii) $(x^2 - 2xy + y^2) - z^2 = (x - y)^2 - (z)^2\ [(a - b)^2 = a^2 - 2ab + b^2]$

$= (x - y - z)\ (x - y + z)\ [a^2 - b^2 = (a - b)\ (a + b)]$

(viii) $25a^2 - 4b^2 + 28bc - 49c^2 = 25a^2 - (4b^2 - 28bc + 49c^2)$

$= (5a)^2 - [(2b)^2 - 2 \times 2b \times 7c + (7c)^2]$

$= (5a)^2 - [(2b - 7c)^2]$

[Using identity $(a - b)^2 = a^2 - 2ab + b^2$]

$= [5a + (2b - 7c)] [5a - (2b - 7c)]$

[Using identity $a^2 - b^2 = (a - b)(a + b)$]

Question 3:Factorise the expressions

(i) $ax^2 + bx$

(ii) $7p^2 + 21q^2$

(iii) $2x^3 + 2xy^2 + 2xz^2$

(iv) $am^2 + bm^2 + bn^2 + an^2$

(v) $(lm + l) + m + 1$

(vi) $y(y + z) + 9(y + z)$

(vii) $5y^2 - 20y - 8z + 2yz$

(viii) $10ab + 4a + 5b + 2$

(ix) $6xy - 4y + 6 - 9x$

(i) $ax^2 + bx = a \times x \times x + b \times x = x(ax + b)$

(ii) $7p^2 + 21q^2 = 7 \times p \times p + 3 \times 7 \times q \times q = 7(p^2 + 3q^2)$

(iii) $2x^3 + 2xy^2 + 2xz^2 = 2x(x^2 + y^2 + z^2)$

(iv)$am^2 + bm^2 + bn^2 + an^2 = am^2 + bm^2 + an^2 + bn^2$

$= m^2(a + b) + n^2(a + b)$

$= (a + b)(m^2 + n^2)$

(v) $(lm + l) + m + 1 = lm + m + l + 1$

$= m(l + 1) + 1(l + 1)$

$= (l + l)(m + 1)$

(vi) $y(y + z) + 9(y + z) = (y + z)(y + 9)$

(vii) $5y^2 - 20y - 8z + 2yz = 5y^2 - 20y + 2yz - 8z$

$= 5y(y - 4) + 2z(y - 4)$

$= (y - 4)\ (5y + 2z)$

(viii) $10ab + 4a + 5b + 2 = 10ab + 5b + 4a + 2$

$= 5b(2a + 1) + 2(2a + 1)$

$= (2a + 1)\ (5b + 2)$

(ix) $6xy - 4y + 6 - 9x = 6xy - 9x - 4y + 6$

$= (5a + 2b - 7c)\ (5a - 2b + 7c)$

Question 4:Factorise

(i) $a^4 - b^4$

(ii) $p^4 - 81$

(iii) $x^4 - (y + z)^4$

(iv) $x^4 - (x - z)^4$

(v) $a^4 - 2a^2b^2 + b^4$

(i) $a^4 - b^4 = (a^2)^2 - (b^2)^2$

$= (a^2 - b^2)\ (a^2 + b^2)$

$= (a - b)\ (a + b)\ (a^2 + b^2)$

(ii) $p^4 - 81 = (p^2)^2 - (9)^2$

$= (p^2 - 9)\ (p^2 + 9)$

$= [(p)^2 - (3)^2]\ (p^2 + 9)$

$= (p - 3)\ (p + 3)\ (p^2 + 9)$

(iii) $x^4 - (y + z)^4 = (x^2)^2 - [(y + z)^2]^2$

$= [x^2 - (y + z)^2]\ [x^2 + (y + z)^2]$

$= [x - (y + z)][\ x + (y + z)]\ [x^2 + (y + z)^2]$

$= (x - y - z)\ (x + y + z)\ [x^2 + (y + z)^2]$

(iv) $x^4 - (x - z)^4 = (x^2)^2 - [(x - z)^2]^2$

$= [x^2 - (x - z)^2]\ [x^2 + (x - z)^2]$

$= [x - (x - z)]\ [x + (x - z)]\ [x^2 + (x - z)^2]$

$= z(2x - z)\ [x^2 + x^2 - 2xz + z^2]$

$= z(2x - z)\ (2x^2 - 2xz + z^2)$

(v) $a^4 - 2a^2b^2 + b^4 = (a^2)^2 - 2\ (a^2)\ (b^2) + (b^2)^2$

$= (a^2 - b^2)^2$

$= [(a - b)\ (a + b)]^2$

$= (a - b)^2\ (a + b)^2$

Question 5:Factorise the following expressions

(i) $p^2 + 6p + 8$

(ii) $q^2 - 10q + 21$

(iii) $p^2 + 6p - 16$

(i) $p^2 + 6p + 8$

It can be observed that, $8 = 4 \times 2$ and $4 + 2 = 6$

$\therefore p^2 + 6p + 8 = p^2 + 2p + 4p + 8$

$= p(p + 2) + 4(p + 2)$

$= (p + 2)\ (p + 4)$

(ii) $q^2 - 10q + 21$

It can be observed that, $21 = (-7) \times (-3)$ and $(-7) + (-3) = -10$

$\therefore q^2 - 10q + 21 = q^2 - 7q - 3q + 21$

$= q(q - 7) - 3(q - 7)$

$= (q - 7)\ (q - 3)$

It can be observed that, $16 = (-2) \times 8$ and $8 + (-2) = 6$

Ex. 13.3

Question 1:Carry out the following divisions.

(i) $28x^4 \div 56x$

(ii) $-36y^3 \div 9y^2$

(iii) $66pq^2r^3 \div 11qr^2$

(iv) $34x^3y^3z^3 \div 51xy^2z^3$

(v) $12a^8b^8 \div (-6a^6b^4)$

(i) $28x^4 = 2 \times 2 \times 7 \times x \times x \times x \times x$

$56x = 2 \times 2 \times 2 \times 7 \times x$

$$28x^4 \div 56x = \frac{2\times2\times7\times x\times x\times x\times x}{2\times2\times2\times7\times x} = \frac{x^3}{2} = \frac{1}{2}x^3$$

(ii) $36y^3 = 2 \times 2 \times 3 \times 3 \times y \times y \times y$

$9y^2 = 3 \times 3 \times y \times y$

$$-36y^3 \div 9y^2 = \frac{-2\times2\times3\times3\times y\times y\times y}{3\times3\times y\times y} = -4y$$

(iii) $66\ pq^2r^3 = 2 \times 3 \times 11 \times p \times q \times q \times r \times r \times r$

$11qr^2 = 11 \times q \times r \times r$

$$66pq^2r^3 \div 11qr^2 = \frac{2\ \times3\times11\times p\times q\times q\times r\times r\times r}{11\times q\times r\times r} = 6pqr$$

(iv) $34x^3y^3z^3 = 2 \times 17 \times x \times x \times x \times y \times y \times y \times z \times z \times z$

$51\ xy^2z^3 = 3 \times 17 \times x \times y \times y \times z \times z \times z$

$$34\ x^3y^3z^3 \div 51\ xy^2z^3 = \frac{2\times17\times x\times x\times x\times y\times y\times y\times z\times z\times z}{3\times17\times x\times y\times y\times z\times z\times z}$$

$$= \frac{2}{3}x^2y$$

(v) $12a^8b^8 = 2 \times 2 \times 3 \times a^8 \times b^8$

$6a^6b^4 = 2 \times 3 \times a^6 \times b^4$

$$12a^8b^8 \div \left(-6a^6b^4\right) = \frac{2\times2\times3\times a^8\times b^8}{-2\times3\times a^6\times b^4} = -2a^2b^4$$

Question 2:Divide the given polynomial by the given monomial.

(i) $(5x^2 - 6x) \div 3x$

(ii) $(3y^8 - 4y^6 + 5y^4) \div y^4$

(iii) $8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 4x^2y^2z^2$

(iv) $(x^3 + 2x^2 + 3x) \div 2x$

(v) $(p^3q^6 - p^6q^3) \div p^3q^3$

(i) $5x^2 - 6x = x(5x - 6)$

$$\left(5x^2 - 6x\right) \div 3x = \frac{x\left(5x - 6\right)}{3x} = \frac{1}{3}\left(5x - 6\right)$$

(ii) $3y^8 - 4y^6 + 5y^4 = y^4(3y^4 - 4y^2 + 5)$

$$\left(3y^8 - 4y^6 + 5y^4\right) \div y^4 = \frac{y^4\left(3y^4 - 4y^2 + 5\right)}{y^4} = 3y^4 - 4y^2 + 5$$

(iii) $8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) = 8x^2y^2z^2(x + y + z)$

$$8\left(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3\right) \div 4x^2y^2z^2 = \frac{8x^2y^2z^2\left(x + y + z\right)}{4x^2y^2z^2} = 2\left(x + y + z\right)$$

(iv) $x^3 + 2x^2 + 3x = x(x^2 + 2x + 3)$

$$\left(x^3 + 2x^2 + 3x\right) \div 2x = \frac{x\left(x^2 + 2x + 3\right)}{2x} = \frac{1}{2}\left(x^2 + 2x + 3\right)$$

(v) $p^3q^6 - p^6q^3 = p^3q^3(q^3 - p^3)$

$$(p^3q^6 - p^6q^3) \div p^3q^3 = \frac{p^3q^3\left(q^3 - p^3\right)}{p^3q^3} = q^3 - p^3$$

Question 3:Work out the following divisions.

(i) $(10x - 25) \div 5$

(ii) $(10x - 25) \div (2x - 5)$

(iii) $10y(6y + 21) \div 5(2y + 7)$

(iv) $9x^2y^2(3z - 24) \div 27xy(z - 8)$

(v) $96abc(3a - 12)(5b - 30) \div 144(a - 4)(b - 6)$

(i) $$(10x-25)\div 5 = \frac{2\times 5\times x-5\times 5}{5} = \frac{5(2x-5)}{5} = 2x-5$$

(ii) $$(10x-25)\div(2x-5) = \frac{2\times 5\times x-5\times 5}{(2x-5)} = \frac{5(2x-5)}{2x-5} = 5$$

(iii) $$10y(6y+21)\div 5(2y+7) = \frac{2\times 5\times y[2\times 3\times y+3\times 7]}{5(2y+7)}$$

$$= \frac{2\times 5\times y\times 3(2y+7)}{5(2y+7)} = 6y$$

(iv) $$9x^2y^2(3z-24)\div 27xy(z-8) = \frac{9x^2y^2[3\times z-2\times 2\times 2\times 3]}{27xy(z-8)}$$

$$= \frac{xy\times 3(z-8)}{3(z-8)} = xy$$

(v) 96 *abc*(3*a* − 12) (5*b* − 30) ÷ 144 (*a* − 4) (*b* − 6)

$$= \frac{96abc(3\times a-3\times 4)(5\times b-2\times 3\times 5)}{144(a-4)(b-6)}$$

$$= \frac{2abc\times 3(a-4)\times 5(b-6)}{3(a-4)(b-6)} = 10abc$$

Question 4:Divide as directed.

(i) 5(2*x* + 1) (3*x* + 5) ÷ (2*x* + 1)

(ii) $26xy(x + 5)\ (y - 4) \div 13x(y - 4)$

(iii) $52pqr\ (p + q)\ (q + r)\ (r + p) \div 104pq(q + r)\ (r + p)$

(iv) $20(y + 4)\ (y^2 + 5y + 3) \div 5(y + 4)$

(v) $x(x + 1)\ (x + 2)\ (x + 3) \div x(x + 1)$

(i) $$5(2x+1)(3x+5) \div (2x+1) = \frac{5(2x+1)(3x+1)}{(2x+1)}$$ $= 5(3x + 1)$

(ii) $$26xy(x+5)(y-4) \div 13x(y-4) = \frac{2 \times 13 \times xy(x+5)(y-4)}{13x(y-4)}$$ $= 2y\ (x + 5)$

(iii) $52pqr\ (p + q)\ (q + r)\ (r + p) \div 104pq\ (q + r)\ (r + p)$

$$= \frac{2 \times 2 \times 13 \times p \times q \times r \times (p+q) \times (q+r) \times (r+p)}{2 \times 2 \times 2 \times 13 \times p \times q \times (q+r) \times (r+p)}$$

$$= \frac{1}{2} r(p+q)$$

(iv) $20(y + 4)\ (y^2 + 5y + 3) = 2 \times 2 \times 5 \times (y + 4)\ (y^2 + 5y + 3)$

$$20(y+4)(y^2+5y+3) \div 5(y+4) = \frac{2 \times 2 \times 5 \times (y+4) \times (y^2+5y+3)}{5 \times (y+4)}$$

$$= 4(y^2+5y+3)$$

(v) $$x(x+1)(x+2)(x+3) \div x(x+1) = \frac{x(x+1)(x+2)(x+3)}{x(x+1)}$$

$= (x + 2)\ (x + 3)$

Question 5:Factorise the expressions and divide them as directed.

(i) $(y^2 + 7y + 10) \div (y + 5)$

(ii) $(m^2 - 14m - 32) \div (m + 2)$

(iii) $(5p^2 - 25p + 20) \div (p - 1)$

(iv) $4yz(z^2 + 6z - 16) \div 2y(z + 8)$

(v) $5pq(p^2 - q^2) \div 2p(p + q)$

(vi) $12xy(9x^2 - 16y^2) \div 4xy(3x + 4y)$

(vii) $39y^3(50y^2 - 98) \div 26y^2(5y + 7)$

(i) $(y^2 + 7y + 10) = y^2 + 2y + 5y + 10$

$= y\,(y + 2) + 5\,(y + 2)$

$= (y + 2)\,(y + 5)$

$$\left(y^2+7y+10\right)\div\left(y+5\right)=\frac{\left(y+5\right)\left(y+2\right)}{\left(y+5\right)}=y+2$$

(ii) $m^2 - 14m - 32 = m^2 + 2m - 16m - 32$

$= m\,(m + 2) - 16\,(m + 2)$

$= (m + 2)\,(m - 16)$

$$\left(m^2-14m-32\right)\div\left(m+2\right)=\frac{\left(m+2\right)\left(m-16\right)}{\left(m+2\right)}=m-16$$

(iii) $5p^2 - 25p + 20 = 5(p^2 - 5p + 4)$

$= 5[p^2 - p - 4p + 4]$

$= 5[p(p - 1) - 4(p - 1)]$

$= 5(p - 1)\,(p - 4)$

$$\left(5p^2-25p+20\right)\div\left(p-1\right)=\frac{5\left(p-1\right)\left(p-4\right)}{\left(p-1\right)}=5\left(p-4\right)$$

(iv) $4yz(z^2 + 6z - 16) = 4yz\,[z^2 - 2z + 8z - 16]$

$= 4yz\,[z(z - 2) + 8(z - 2)]$

$= 4yz(z - 2)\,(z + 8)$

$$4yz\left(z^2+6z-16\right)\div 2y\left(z+8\right)=\frac{4yz\left(z-2\right)\left(z+8\right)}{2y\left(z+8\right)}=2z\left(z-2\right)$$

(v) $5pq(p^2 - q^2) = 5pq\,(p - q)\,(p + q)$

$$5pq\left(p^2-q^2\right)\div 2p\left(p+q\right)=\frac{5pq\left(p-q\right)\left(p+q\right)}{2p\left(p+q\right)}=\frac{5}{2}q\left(p-q\right)$$

(vi) $12xy(9x^2 - 16y^2) = 12xy[(3x)^2 - (4y)^2] = 12xy(3x - 4y)\ (3x + 4y)$

$$12xy\left(9x^2-16y^2\right)\div 4xy\left(3x+4y\right)=\frac{2\times 2\times 3\times x\times y\times\left(3x-4y\right)\times\left(3x+4y\right)}{2\times 2\times x\times y\times\left(3x+4y\right)}$$
$$=3\left(3x-4y\right)$$

(vii) $39y^3(50y^2 - 98) = 3 \times 13 \times y \times y \times y \times 2[(25y^2 - 49)]$

$= 3 \times 13 \times 2 \times y \times y \times y \times [(5y)^2 - (7)^2]$

$= 3 \times 13 \times 2 \times y \times y \times y\ (5y - 7)\ (5y + 7)$

$26y^2(5y + 7) = 2 \times 13 \times y \times y \times (5y + 7)$

$39y^3(50y^2 - 98) \div 26y^2\ (5y + 7)$

Exercise 13.4

Question 1:Find and correct the errors in the statement: 4(x − 5) = 4x − 5

L.H.S. $= 4(x - 5) = 4 \times x - 4 \times 5 = 4x - 20 \neq$ R.H.S.

The correct statement is $4(x - 5) = 4x - 20$

Question 2:Find and correct the errors in the statement: x(3x + 2) = 3x2 + 2

L.H.S. $= x(3x + 2) = x \times 3x + x \times 2 = 3x^2 + 2x \neq$ R.H.S.

The correct statement is $x(3x + 2) = 3x^2 + 2x$

Question 3:Find and correct the errors in the statement: 2x + 3y = 5xy

L.H.S $= 2x + 3y \neq$ R.H.S.

The correct statement is $2x + 3y = 2x + 3y$

Question 4:Find and correct the errors in the statement: x + 2x + 3x = 5x

L.H.S $= x + 2x + 3x = 1x + 2x + 3x = x(1 + 2 + 3) = 6x \neq$ R.H.S.

The correct statement is $x + 2x + 3x = 6x$

Question 5:Find and correct the errors in the statement: 5y + 2y + y − 7y = 0

L.H.S. = $5y + 2y + y - 7y = 8y - 7y = y \neq$ R.H.S

The correct statement is $5y + 2y + y - 7y = y$

Question 6:Find and correct the errors in the statement: 3x + 2x = 5x2

L.H.S. = $3x + 2x = 5x \neq$ R.H.S

The correct statement is $3x + 2x = 5x$

Question 7:Find and correct the errors in the statement: (2x)2 + 4(2x) + 7 = 2x2 + 8x + 7

L.H.S = $(2x)^2 + 4(2x) + 7 = 4x^2 + 8x + 7 \neq$ R.H.S

The correct statement is $(2x)^2 + 4(2x) + 7 = 4x^2 + 8x + 7$

Question 8:

Find and correct the errors in the statement: $(2x)^2 + 5x = 4x + 5x = 9x$

L.H.S = $(2x)^2 + 5x = 4x^2 + 5x \neq$ R.H.S.

The correct statement is $(2x)^2 + 5x = 4x^2 + 5x$

Question 9:Find and correct the errors in the statement: (3x + 2)2 = 3x2 + 6x + 4

L.H.S. = $(3x + 2)^2 = (3x)^2 + 2(3x)(2) + (2)^2$ $[(a + b)^2 = a^2 + 2ab + b^2]$

$= 9x^2 + 12x + 4 \neq$ R.H.S

The correct statement is $(3x + 2)^2 = 9x^2 + 12x + 4$

Question 10:Find and correct the errors in the following mathematical statement. Substituting x = −3 in

(a) $x^2 + 5x + 4$ gives $(-3)^2 + 5(-3) + 4 = 9 + 2 + 4 = 15$

(b) $x^2 - 5x + 4$ gives $(-3)^2 - 5(-3) + 4 = 9 - 15 + 4 = -2$

(c) $x^2 + 5x$ gives $(-3)^2 + 5(-3) = -9 - 15 = -24$

(a) For $x = -3$,

$x^2 + 5x + 4 = (-3)^2 + 5(-3) + 4 = 9 - 15 + 4 = 13 - 15 = -2$

(b) For $x = -3$,

$x^2 - 5x + 4 = (-3)^2 - 5(-3) + 4 = 9 + 15 + 4 = 28$

(c) For $x = -3$,

$x^2 + 5x = (-3)^2 + 5(-3) = 9 - 15 = -6$

Question 11:Find and correct the errors in the statement: (y − 3)2 = y2 − 9

L.H.S = $(y - 3)^2 = (y)^2 - 2(y)(3) + (3)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$

$= y^2 - 6y + 9 \neq$ R.H.S

The correct statement is $(y - 3)^2 = y^2 - 6y + 9$

Question 12:Find and correct the errors in the statement: (z + 5)2 = z2 + 25

L.H.S = $(z + 5)^2 = (z)^2 + 2(z)(5) + (5)^2$ $[(a + b)^2 = a^2 + 2ab + b^2]$

$= z^2 + 10z + 25 \neq$ R.H.S

The correct statement is $(z + 5)^2 = z^2 + 10z + 25$

Question 13:

Find and correct the errors in the statement: $(2a + 3b)(a - b) = 2a^2 - 3b^2$

L.H.S. = $(2a + 3b)(a - b) = 2a \times a + 3b \times a - 2a \times b - 3b \times b$

$= 2a^2 + 3ab - 2ab - 3b^2 = 2a^2 + ab - 3b^2 \neq$ R.H.S.

The correct statement is $(2a + 3b)(a - b) = 2a^2 + ab - 3b^2$

Question 14:Find and correct the errors in the statement: (a + 4) (a + 2) = a2 + 8

L.H.S. = $(a + 4)(a + 2) = (a)^2 + (4 + 2)(a) + 4 \times 2$

$= a^2 + 6a + 8 \neq$ R.H.S

The correct statement is $(a + 4)(a + 2) = a^2 + 6a + 8$

Question 15:Find and correct the errors in the statement: (a − 4) (a − 2) = a2 − 8

L.H.S. = $(a - 4)(a - 2) = (a)^2 + [(-4) + (-2)](a) + (-4)(-2)$

$= a^2 - 6a + 8 \neq$ R.H.S.

The correct statement is $(a - 4)(a - 2) = a^2 - 6a + 8$

Question 16:Find and correct the errors in the statement: $\frac{3x^2}{3x^2} = 0$

L.H.S = $\frac{3x^2}{3x^2} = \frac{3 \times x \times x}{3 \times x \times x} = 1 \neq \text{R.H.S.}$

The correct statement is $\frac{3x^2}{3x^2} = 1$

Question 17:Find and correct the errors in the statement: $\frac{3x^2+1}{3x^2} = 1+1 = 2$

$\frac{3x^2+1}{3x^2} = \frac{3x^2}{3x^2} + \frac{1}{3x^2} = 1 + \frac{1}{3x^2} \neq \text{R.H.S.}$

The correct statement is $\frac{3x^2+1}{3x^2} = 1 + \frac{1}{3x^2}$

Question 18:Find and correct the errors in the statement: $\frac{3x}{3x+2} = \frac{1}{2}$

L.H.S = $\frac{3x}{3x+2} \neq \text{R.H.S.}$

The correct statement is $\frac{3x}{3x+2} = \frac{3x}{3x+2}$

Question 19:Find and correct the errors in the statement: $\frac{3}{4x+3} = \frac{1}{4x}$

L.H.S. = $\frac{3}{4x+3}$ ≠ R.H.S.

The correct statement is $\frac{3}{4x+3} = \frac{3}{4x+3}$

Question 20:Find and correct the errors in the statem $\frac{4x+5}{4x} = 5$

L.H.S. = $\frac{4x+5}{4x} = \frac{4x}{4x} + \frac{5}{4x} = 1 + \frac{5}{4x} \neq \text{R.H.S}$

The correct statement is $\frac{4x+5}{4x} = 1 + \frac{5}{4x}$

Question 21:Find and correct the errors in the statement:

$\frac{7x+5}{5} = 7x$

L.H.S. =

The correct statement is $\frac{7x+5}{5} = \frac{7x}{5} + 1$

$$\frac{7x+5}{5} = \frac{7x}{5} + \frac{5}{5} = \frac{7x}{5} + 1 \neq \text{R.H.S.}$$

What Have we Discussed

1. When we factorise an expression, we write it as a product of factors. These factors may be numbers, algebraic variables or algebraic expressions.
2. An irreducible factor is a factor which cannot be expressed further as a product of factors.
3. A systematic way of factorising an expression is the common factor method. It consists of three steps: (i) Write each term of the expression as a product of irreducible factors (ii) Look for and separate the common factors and (iii) Combine the remaining factors in each term in accordance with the distributive law.
4. Sometimes, all the terms in a given expression do not have a common factor; but the terms can be grouped in such a way that all the terms in each group have a common factor. When we do this, there emerges a common factor across all the groups leading to the required factorisation of the expression. This is the method of regrouping.
5. In factorisation by regrouping, we should remember that any regrouping (i.e., rearrangement) of the terms in the given expression may not lead to factorisation. We must observe the expression and come out with the desired regrouping by trial and error.
6. A number of expressions to be factorised are of the firm or can be put into the form $a^2 + 2ab + b^2$, $a^2 - 2ab + b^2$, $a^2 - b^2$ and $x^2 + (a + b) + ab$. These expressions can be easily factorised using *Identities I, II, III and IV, given in Chapter 9,*

   $a^2 + 2ab + b^2 = (a+b)^2$

   $a^2 - 2ab + b^2 = (a - b)^2$

   $a^2 - b^2 = (a + b)(a-b)$

   $x^2 + (a + b)x + ab = (x + a)(x + b)$
7. In expressions which have factors of the type $(x + a)(x + b)$, remember the numerical term gives ab. Its factors, a and b, should be so chosen that their sum, with signs taken care of, is the coefficient of x.
8. We know that in the case of numbers, division is the inverse of multiplication. This idea is applicable also to the division of algebraic expressions.
9. In the case of division of a polynomial by a monomial, we may carry out the division either by dividing each term of the polynomial by the monomial or by the common factor method.
10. In the case of division of a polynomial by a polynomial, we cannot proceed by dividing each term in the dividend polynomial by the divisor polynomial. Instead, we factorise both the polynomials and cancel their common factors.

11. In the case of divisions of algebraic expressions that we studied in this chapter, we have Dividend = Divisor x Quotient.
In general, however, the relation is Dividend = Divisor x Quotient +Remainder
Thus, we have considered in the present chapter only those divisions in which the remainder is zero.
12. There are many errors students commonly make when solving algebra exercises. You should avoid making such errors.