

Direct and Inverse Proportions (Math)

Exercise 12.1

Question 1: Following are the car parking charges near a railway station up to

4 hours Rs 60

8 hours Rs 100

12 hours Rs 140

24 hours Rs 180

Check if the parking charges are in direct proportion to the parking time.

A table of the given information is formed as

Number of hours	4	8	12	24
Parking charges (in Rs)	60	100	140	180

The ratio of parking charges to the respective number of hours (Rs/ hour) can be calculated as

$$\frac{60}{4} = 15, \quad \frac{100}{8} = \frac{25}{2}, \quad \frac{140}{12} = \frac{35}{3}, \quad \frac{180}{24} = \frac{15}{2}$$

As each ratio is not same, therefore, the parking charges are not in a direct proportion to the parking time.

Question 2: A mixture of paint is prepared by mixing 1 part of red pigments with 8 parts of base. In the following table, find the parts of base that need to be added.

Parts of red pigment	1	4	7	12	20
parts of base	8

The given mixture of paint is prepared by mixing 1 part of red pigments with 8 parts of base. For more parts of red pigments, the parts of the base will also be more. Therefore, the parts of red pigments and the parts of base are in direct proportion. The given information in the form of a table is as follows.

Parts of red pigment	1	4	7	12	20
Parts of base	8	x_1	x_2	x_3	x_4

According to direct proportion,

$$\frac{x_1}{4} = \frac{8}{1} \Rightarrow x_1 = 4 \times 8 = 32$$

$$\frac{x_2}{7} = \frac{8}{1} \Rightarrow x_2 = 7 \times 8 = 56$$

$$\frac{x_3}{12} = \frac{8}{1} \Rightarrow x_3 = 8 \times 12 = 96$$

$$\frac{x_4}{20} = \frac{8}{1} \Rightarrow x_4 = 8 \times 20 = 160$$

The table can be drawn as follows.

Parts of red pigment	1	4	7	12	20
Parts of base	8	32	56	96	160

Question 3: In Question 2 above, if 1 part of a red pigment requires 75 mL of base, how much red pigment should we mix with 1800 mL of base?

Let the parts of red pigment required to mix with 1800 mL of base be x .

The given information in the form of a table is as follows.

Parts of red pigment	1	x
Parts of base (in mL)	75	1800

The parts of red pigment and the parts of base are in direct proportion.

Therefore, we obtain

$$\begin{aligned}\frac{1}{75} &= \frac{x}{1800} \\ \Rightarrow x &= \frac{1 \times 1800}{75} \\ \Rightarrow x &= 24\end{aligned}$$

Thus, 24 parts of red pigments should be mixed with 1800 mL of base.

Question 4: A machine in a soft drink factory fills 840 bottles in six hours. How many bottles will it fill in five hours?

Let the number of bottles filled by the machine in five hours be x .

The given information in the form of a table is as follows.

Number of bottles	840	x
Time taken (in hours)	6	5

The number of bottles and the time taken to fill these bottles are in direct proportion. Therefore, we obtain

$$\frac{840}{6} = \frac{x}{5}$$

$$x = \frac{840 \times 5}{6} = 700$$

Thus, 700 bottles will be filled in 5 hours.

Question 5: A photograph of a bacteria enlarged 50,000 times attains a length of 5 cm. What is the actual length of the bacteria? If the photograph is enlarged 20,000 times only, what would be its enlarged length?

Let the actual length of bacteria be x cm and the enlarged length of bacteria be y cm, if the photograph is enlarged for 20,000 times.

The given information in the form of a table is as follows.

Length of bacteria (in cm)	5	x	y
Number of times photograph of Bacteria was enlarged	50000	1	20000

The number of times the photograph of bacteria was enlarged and the length of bacteria are in direct proportion.

Therefore, we obtain

$$\frac{5}{50,000} = \frac{x}{1}$$

$$\Rightarrow x = \frac{1}{10000} = 10^{-4}$$

Hence, the actual length of bacteria is 10^{-4} cm.

Let the length of bacteria when the photograph of bacteria is enlarged 20, 000 times be y .

$$\frac{5}{50,000} = \frac{y}{20,000}$$

$$y = \frac{20,000 \times 5}{50,000} = 2$$

Hence, the enlarged length of bacteria is 2 cm.

Question 6: In a model of a ship, the mast is 9 cm high, while the mast of the actual ship is 12 m high. If the length of the ship is 28 m, how long is the model ship?

Let the length of the mast of the model ship be x cm.

The given information in the form of a table is as follows:

-	Height of mast	Length of ship
Model ship	9 cm	x
Actual ship	12 m	28 m

We know that the dimensions of the actual ship and the model ship are directly proportional to each other.

Therefore, we obtain:

$$\frac{12}{9} = \frac{28}{x}$$

$$x = \frac{28 \times 9}{12} = 21$$

Thus, the length of the model ship is 21 cm.

Question 7: Suppose 2 kg of sugar contains 9×10^6 crystals.

How many sugar crystals are there in (i) 5 kg of sugar? (ii) 1.2 kg of sugar?

(i) Let the number of sugar crystals in 5 kg of sugar be x .

The given information in the form of a table is as follows.

Amount of sugar (in kg)	2	5
Number of crystals	9×10^6	x

The amount of sugar and the number of crystals it contains are directly proportional to each other. Therefore, we obtain

$$\frac{2}{9 \times 10^6} = \frac{5}{x}$$

$$x = \frac{5 \times 9 \times 10^6}{2} = 2.25 \times 10^7$$

Hence, the number of sugar crystals is 2.25×10^7 .

(ii) Let the number of sugar crystals in 1.2 kg of sugar be y . The given information in the form of a table is as follows.

Amount of sugar (in kg)	2	1.2
Number of crystals	9×10^6	y

$$\frac{2}{9 \times 10^6} = \frac{1.2}{y}$$

$$y = \frac{1.2 \times 9 \times 10^6}{2} = 5.4 \times 10^6$$

Hence, the number of sugar crystals is 5.4×10^6 .

Question 8: Razia has a road map with a scale of 1 cm representing 18 km. She drives on a road for 72 km. What would be her distance covered in the map?

Let the distance represented on the map be x cm.

The given information in the form of a table is as follows.

Distance covered on road in (in km)	18	72
Distance represented on map (in cm)	1	x

The distances covered on road and represented on map are directly proportional to each other. Therefore, we obtain

$$\frac{18}{1} = \frac{72}{x}$$

$$\Rightarrow x = \frac{72}{18} = 4$$

Hence, the distance represented on the map is 4 cm.

Question 9: A 5 m 60 cm high vertical pole casts a shadow 3 m 20 cm long. Find at the same time.

(i) the length of the shadow cast by another pole 10 m 50 cm high

(ii) the height of a pole which casts a shadow 5 m long.

(i) Let the length of the shadow of the other pole be x m.

$$1 \text{ m} = 100 \text{ cm}$$

The given information in the form of a table is as follows.

Height of pole (in m)	5.60	10.50
Length of shadow (in m)	3.20	x

More the height of an object, more will be the length of its shadow.

Thus, the height of an object and length of its shadow are directly proportional to each other. Therefore, we obtain

$$\begin{aligned}\frac{5.60}{3.20} &= \frac{10.50}{x} \\ \Rightarrow x &= \frac{10.50 \times 3.20}{5.60} = 6\end{aligned}$$

Hence, the length of the shadow will be 6 m.

(ii) Let the height of the pole be y m.

The given information in the form of a table is as follows.

Height of pole (in m)	5.60	y
Length of shadow (in m)	3.20	5

The height of the pole and the length of the shadow are directly proportional to each other. Therefore,

$$\begin{aligned}\frac{5.60}{3.20} &= \frac{y}{5} \\ y &= \frac{5 \times 5.60}{3.20} = 8.75\end{aligned}$$

Thus, the height of the pole is 8.75 m or 8 m 75 cm.

Question 10: A loaded truck travels 14 km in 25 minutes. If the speed remains the same, how far can it travel in 5 hours?

Let the distance travelled by the truck in 5 hours be x km.

We know, 1 hour = 60 minutes

\therefore 5 hours = (5×60) minutes = 300 minutes

The given information in the form of a table is as follows.

Distance travelled (in km)	14	x
Time (in min)	25	300

The distance travelled by the truck and the time taken by the truck are directly proportional to each other. Therefore,

$$\frac{14}{25} = \frac{x}{300}$$
$$x = \frac{14 \times 300}{25} = 168$$

Hence, the distance travelled by the truck is 168 km.

Ex. 12.2

Question 1: Which of the following are in inverse proportion?

- (i) The number of workers on a job and the time to complete the job.
- (ii) The time taken for a journey and the distance travelled in a uniform speed.
- (iii) Area of cultivated land and the crop harvested.
- (iv) The time taken for a fixed journey and the speed of the vehicle.
- (v) The population of a country and the area of land per person.

(i) These are in inverse proportion because if there are more workers, then it will take lesser time to complete that job.

(ii) No, these are not in inverse proportion because in more time, we may cover more distance with a uniform speed.

(iii) No, these are not in inverse proportion because in more area, more quantity of crop may be harvested.

(iv) These are in inverse proportion because with more speed, we may complete a certain distance in a lesser time.

(v) These are in inverse proportion because if the population is increasing, then the area of the land per person will be decreasing accordingly.

Question 2: In a Television game show, the prize money of Rs 1,00,000 is to be divided equally amongst the winners. Complete the following table and find whether the prize money given to an individual winner is directly or inversely proportional to the number of winners?

Number of winners	1	2	4	5	8	10	20
Prize for each winner (in Rs)	100000	50000

A table of the given information is as follows.

Number of winners	1	2	4	5	8	10	20
Prize for each winner (in Rs)	100000	50000	x_1	x_2	x_3	x_4	x_5

From the table, we obtain

$$1 \times 100000 = 2 \times 50000 = 100000$$

Thus, the number of winners and the amount given to each winner are inversely proportional to each other. Therefore,

$$1 \times 100000 = 4 \times x_1$$

$$x_1 = \frac{100000}{4} = 25000$$

$$1 \times 100000 = 5 \times x_2$$

$$x_2 = \frac{100000}{5} = 20000$$

$$1 \times 100000 = 8 \times x_3$$

$$x_3 = \frac{100000}{8} = 12500$$

$$1 \times 100000 = 10 \times x_4$$

$$x_4 = \frac{100000}{10} = 10000$$

$$1 \times 100000 = 20 \times x_5$$

$$x_5 = \frac{100000}{20} = 5000$$

Question 3: Raheem is making a wheel using spokes. He wants to fix equal spokes in such a way that the angles between any pair of consecutive spokes are equal. Help him by completing the following table.

Number of spokes	4	6	8	10	12
Angle between a pair of consecutive spokes	90°	60°

(i) Are the number of spokes and the angles formed between the pairs of consecutive spokes in inverse proportion?

(ii) Calculate the angle between a pair of consecutive spokes on a wheel with 15 spokes.

(iii) How many spokes would be needed, if the angle between a pair of consecutive spokes is 40°?

A table of the given information is as follows.

Number of spokes	4	6	8	10	12
Angle between a pair of consecutive spokes	90°	60°	x_1	x_2	x_3

From the given table, we obtain

$$4 \times 90^\circ = 360^\circ = 6 \times 60^\circ$$

Thus, the number of spokes and the angle between a pair of consecutive spokes are inversely proportional to each other. Therefore,

$$4 \times 90^\circ = x_1 \times 8$$

$$x_1 = \frac{4 \times 90^\circ}{8} = 45^\circ$$

Similarly, $x_2 = \frac{4 \times 90^\circ}{10} = 36^\circ$ and $x_3 = \frac{4 \times 90^\circ}{12} = 30^\circ$

Thus, the following table is obtained.

Number of spokes	4	6	8	10	12
Angle between a pair of consecutive spokes	90°	60°	45°	36°	30°

(i) Yes, the number of spokes and the angles formed between the pairs of consecutive spokes are in inverse proportion.

(ii) Let the angle between a pair of consecutive spokes on a wheel with 15 spokes be x . Therefore,

$$4 \times 90^\circ = 15 \times x$$

$$x = \frac{4 \times 90^\circ}{15} = 24^\circ$$

Hence, the angle between a pair of consecutive spokes of a wheel, which has 15 spokes in it, is 24° .

(iii) Let the number of spokes in a wheel, which has 40° angles between a pair of consecutive spokes, be y .

Therefore,

$$4 \times 90^\circ = y \times 40^\circ$$

$$y = \frac{4 \times 90}{40} = 9$$

Hence, the number of spokes in such a wheel is 9.

Question 4: If a box of sweets is divided among 24 children, they will get 5 sweets each. How many would each get, if the number of the children is reduced by 4?

$$\text{Number of remaining children} = 24 - 4 = 20$$

Let the number of sweets which each of the 20 students will get, be x .

The following table is obtained.

Number of students	24	20
Number of sweets	5	x

If the number of students is lesser, then each student will get more number of sweets.

Since this is a case of inverse proportion,

$$24 \times 5 = 20 \times x$$

$$x = \frac{24 \times 5}{20} = 6$$

Hence, each student will get 6 sweets.

Question 5: A farmer has enough food to feed 20 animals in his cattle for 6 days. How long would the food last if there were 10 more animals in his cattle?

Let the number of days that the food will last if there were 10 more animals in the cattle be x . The following table is obtained.

Number of animals	20	$20 + 10 = 30$
Number of days	6	x

More the number of animals, lesser will be the number of days for which the food will last.

Hence, the number of days the food will last and the number of animals are inversely proportional to each other.

Therefore,

$$20 \times 6 = 30 \times x$$

$$x = \frac{20 \times 6}{30} = 4$$

Thus, the food will last for 4 days.

Question 6: A contractor estimates that 3 persons could rewire Javaid's house in 4 days. If, he uses 4 persons instead of three, how long should they take to complete the job?

Let the number of days required by 4 persons to complete the job be x .

The following table is obtained.

Number of days	4	x
Number of persons	3	4

If the number of persons is more, then it will take lesser time to complete the job.

Hence, the number of days and the number of persons required to complete the job are inversely proportional to each other.

Therefore,

$$4 \times 3 = x \times 4$$

$$x = \frac{4 \times 3}{4} = 3$$

Thus, the number of days required to complete the job is 3.

Question 7: A batch of bottles was packed in 25 boxes with 12 bottles in each box. If the same batch is packed using 20 bottles in each box, how many boxes would be filled?

Let the number of boxes filled, by using 20 bottles in each box, be x .

The following table is obtained.

Number of bottles	12	20
Number of boxes	25	x

More the number of bottles, lesser will be the number of boxes.

Hence, the number of bottles and the number of boxes required to pack these are inversely proportional to each other.

Therefore,

$$12 \times 25 = 20 \times x$$

$$x = \frac{12 \times 25}{20} = 15$$

Hence, the number of boxes required to pack these bottles is 15.

Question 8: A factory required 42 machines to produce a given number of articles in 63 days. How many machines would be required to produce the same number of articles in 54 days?

Let the number of machines required to produce articles in 54 days be x . The following table is obtained.

Number of machines	42	x
Number of days	63	54

More the number of machines, lesser will be the number of days that it will take to produce the given number of articles. Thus, this is a case of inverse proportion. Therefore,

$$42 \times 63 = 54 \times x$$

$$x = \frac{42 \times 63}{54} = 49$$

Hence, the required number of machines to produce the given number of articles in 54 days is 49.

Question 9: A car takes 2 hours to reach a destination by travelling at the speed of 60 km/h. how long will it.

Let the time taken by the car to reach the destination, while travelling with a speed of 80 km/hr, be x hours.

The following table is obtained.

Speed (in km/hr)	60	80
Time taken (in hours)	2	x

More the speed of the car, lesser will be the time taken by it to reach the destination.

Hence, the speed of the car and the time taken by the car are inversely proportional to each other. Therefore,

$$60 \times 2 = 80 \times x$$

$$x = \frac{60 \times 2}{80} = \frac{3}{2} = 1\frac{1}{2}$$

The time required by the car to reach the given destination is $1\frac{1}{2}$ hours.

Question 10: Two persons could fit new windows in house in 3 days.

(i) One of the persons fell ill before the work started. How long would the job take now?

(ii) How many persons would be needed to fit the windows in one day?

(i) Let the number of days required by 1 man to fit all the windows be x . The following table is obtained.

Number of persons	2	1
Number of days	3	x

Lesser the number of persons, more will be the number of days required to fit all the windows. Hence, this is a case of inverse proportion. Therefore,

$$2 \times 3 = 1 \times x$$

$$x = 6$$

Hence, the number of days taken by 1 man to fit all the windows is 6.

(ii) Let the number of persons required to fit all the windows in one day be y . The following table is formed.

Number of persons	2	y
Number of days	3	1

Lesser the number of days, more will be the number of persons required to fit all the windows. Hence, this is a case of inverse proportion. Therefore,

$$2 \times 3 = y \times 1$$

$$y = 6$$

Hence, 6 persons are required to fit all the windows in one day.

Question 11: A school has 8 periods a day each of 45 minutes duration. How long would each period be, if the school has 9 periods a day, assuming the number of school hours to be the same?

Let the duration of each period, when there are 9 periods a day in the school, be x minutes. The following table is obtained.

Duration of each period (in minutes)	45	x
Number of periods	8	9

If there is more number of periods a day in the school, then the duration of each period will be lesser. Hence, this is a case of inverse proportion. Therefore

$$45 \times 8 = x \times 9$$

$$x = \frac{45 \times 8}{9} = 40$$

Hence, in this case, the duration of each period will be 40 minutes.

What Have we Discussed

1. Two quantities x and y are said to be in direct proportion if they increase (decrease) together in such a manner that the ratio of their corresponding values remains constant.
2. Two quantities x and y are said to be in inverse proportion if an increase in x causes a proportional decrease in y (and vice-versa) in such a manner that the product of their corresponding values remains constant. That is, if $xy = k$, then x and y are said to vary inversely.