Exponents and Powers (Math)

Exercise 11.1

Question 1:Evaluate

(i) 3^{-2} (ii) $(-4)^{-2}$ (iii) $\left(\frac{1}{2}\right)^{-5}$

(i) $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ $\left(a^{-m} = \frac{1}{a^m}\right)$

(ii) $(-4)^{-2} = \frac{1}{(-4)^2} = \frac{1}{16}$ $\left(a^{-m} = \frac{1}{a^m}\right)$

(iii) $\left(\frac{1}{2}\right)^{-5} = \frac{1}{(2)^{-5}} = (2)^5 = 2\times2\times2\times2\times2 = 32$

Question 2:Simplify and express the result in power notation with positive exponent.

(i) $(-4)^5 \div (-4)^8$ (ii) $\left(\frac{1}{2^3}\right)^2$

(iii) $(-3)^4 \times \left(\frac{5}{3}\right)^4$ (iv) $\left(3^{-7} \div 3^{-10}\right) \times 3^{-5}$

(v) $2^{-3} \times (-7)^{-3}$

(i) $(-4)^5 \div (-4)^8 = (-4)^{5-8}$ ($a^m \div a^n = a^{m-n}$)

$= (-4)^{-3}$

$= \frac{1}{(-4)^3}$ $\left(a^{-m} = \frac{1}{a^m}\right)$

(ii) $\left(\frac{1}{2^3}\right)^2 = \frac{1}{\left(2^3\right)^2} = \frac{1}{2^6}$ $\left(\left(a^m\right)^n = a^{mn}\right)$

(iii) $\left(-3\right)^4 \times \left(\frac{5}{3}\right)^4 = \left(-1\times 3\right)^4 \times \frac{5^4}{3^4}$

$= \left(-1\right)^4 \times 3^4 \times \frac{5^4}{3^4}$ $\left[\left(ab\right)^m = a^m \times b^m\right]$

$= \left(-1\right)^4 \times 5^4$

$= 5^4$ $\left[\left(-1\right)^4 = 1\right]$

(iv) $(3^{-7} \div 3^{-10}) \times 3^{-5} = (3^{-7-(-10)}) \times 3^{-5}$ ($a^m \div a^n = a^{m-n}$)

$= 3^3 \times 3^{-5}$

$= 3^{3+(-5)}$ ($a^m \times a^n = a^{m+n}$)

$= 3^{-2}$

$= \frac{1}{3^2}$ $\left(a^{-m} = \frac{1}{a^m}\right)$

(v) $2^{-3} \times (-7)^{-3} = \frac{1}{2^3} \times \frac{1}{\left(-7\right)^3}$ $\left(a^{-m} = \frac{1}{a^m}\right)$

$= \frac{1}{\left[2\times\left(-7\right)\right]^3}$ $\left[a^m \times b^m = \left(ab\right)^m\right]$

$= \frac{1}{\left(-14\right)^3}$

Question 3:Find the value of.

(i) $(3^0 + 4^{-1}) \times 2^2$ (ii) $(2^{-1} \times 4^{-1}) \div 2^{-2}$

(ii) (iv) $(3^{-1} + 4^{-1} + 5^{-1})^0$

(i) $(2^{-1} \times 4^{-1}) \div 2^{-2} = [2^{-1} \times \{(2)^2\}^{-1}] \div 2^{-2}$

$= (2^{-1} \times 2^{-2}) \div 2^{-2}$

$= 2^{-1+(-2)} \div 2^{-2}$ ($a^m \times a^n = a^{m+n}$)

$= 2^{-3} \div 2^{-2}$

$= 2^{-3-(-2)}$ ($a^m \div a^n = a^{m-n}$)

$= 2^{-3+2} = 2^{-1}$

(ii) $(3^{-1} + 4^{-1} + 5^{-1})^0$

$= 1$ ($a^0 = 1$)

Question 4:Evaluate (i) $\frac{8^{-1} \times 5^3}{2^{-4}}$ **(ii)** $\left(5^{-1} \times 2^{-1}\right) \times 6^{-1}$

(i) $\frac{8^{-1} \times 5^3}{2^{-4}} = \frac{2^4 \times 5^3}{8^1}$ $\left(a^{-m} = \frac{1}{a^m}\right)$

$= \frac{2^4 \times 5^3}{2^3} = 2^{4-3} \times 5^3$ $\left(a^m \div a^n = a^{m-n}\right)$

$= 2 \times 125 = 250$

(ii) $(5^{-1} \times 2^{-1}) \times 6^{-1} = \left(\frac{1}{5} \times \frac{1}{2}\right) \times \frac{1}{6}$ $\left(a^{-m} = \frac{1}{a^m}\right)$

$= \frac{1}{10} \times \frac{1}{6} = \frac{1}{60}$

Question 5:Find the value of m for which 5m ÷5−3 = 55.

$5^m \div 5^{-3} = 5^5$

$5^{m-(-3)} = 5^5$ ($a^m \div a^n = a^{m-n}$)

$5^{m+3} = 5^5$

Since the powers have same bases on both sides, their respective exponents must be equal.

$m + 3 = 5$

$m = 5 - 3$

$m = 2$

Question 6:Evaluate (i) $\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1}$ **(ii)** $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-4}$

(i) $\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1} = \left\{\left(\frac{3}{1}\right)^{1} - \left(\frac{4}{1}\right)^{1}\right\}^{-1} \qquad \left(a^{-m} = \frac{1}{a^m}\right)$

$= \{3-4\}^{-1} = (-1)^{-1} = \frac{1}{-1} = -1$

(ii) $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-4} = \frac{5^{-7}}{8^{-7}} \times \frac{8^{-4}}{5^{-4}} \qquad \left[\left(\frac{a}{b}\right)^{m} = \frac{a^m}{b^m}\right]$

$= \frac{8^7}{5^7} \times \frac{5^4}{8^4} \qquad \left(a^{-m} = \frac{1}{a^m}\right)$

$= \frac{8^{7-4}}{5^{7-4}} \qquad \left(a^m \div a^n = a^{m-n}\right)$

$= \frac{8^3}{5^3} = \frac{512}{125}$

Question 7:Simplify. (i) $\frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}} (t \neq 0)$ **(ii)** $\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$

(i) $\frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}} = \frac{5^2 \times t^{-4}}{5^{-3} \times 5 \times 2 \times t^{-8}}$

$$= \frac{5^2 \times t^{-4}}{5^{-3+1} \times 2 \times t^{-8}} \qquad \left(a^m \times a^n = a^{m+n}\right)$$

$$= \frac{5^2 \times t^{-4}}{5^{-2} \times 2 \times t^{-8}}$$

$$= \frac{5^{2-(-2)} t^{-4-(-8)}}{2} \qquad \left(a^m \div a^n = a^{m-n}\right)$$

$$= \frac{5^4 t^4}{2} = \frac{625\, t^4}{2}$$

(ii)
$$\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}} = \frac{3^{-5} \times (2 \times 5)^{-5} \times 5^3}{5^{-7} \times (2 \times 3)^{-5}}$$

$$= \frac{3^{-5} \times 2^{-5} \times 5^{-5} \times 5^3}{5^{-7} \times 2^{-5} \times 3^{-5}} \qquad \left[(a \times b)^m = a^m \times b^m\right]$$

$$= 3^{-5-(-5)} \times 2^{-5-(-5)} \times 5^{-5+3-(-7)} \qquad \left(a^m \div a^n = a^{m-n}\right)$$

$$= 3^0 \times 2^0 \times 5^5 \qquad \left(a^0 = 1\right)$$

$$= 5^5$$

Exercise 11.2

Question 1:Express the following numbers in standard form.

(i) 0.0000000000085 (ii) 0.00000000000942

(iii) 6020000000000000 (iv) 0.00000000837

(v) 31860000000

(i) 0.0000000000085 = 8.5×10^{-12}

(ii) 0.00000000000942 = 9.42×10^{-12}

(iii) 6020000000000000 = 6.02×10^{15}

(iv) 0.00000000837 = 8.37×10^{-9}

(v) 31860000000 = 3.186×10^{10}

Question 2:Express the following numbers in usual form.

(i) 3.02×10^{-6} (ii) 4.5×10^{4}

(iii) 3×10^{-8} (iv) 1.0001×10^{9}

(v) 5.8×10^{12} (vi) 3.61492×10^{6}

(i) $3.02 \times 10^{-6} = 0.00000302$

(ii) $4.5 \times 10^{4} = 45000$

(iii) $3 \times 10^{-8} = 0.00000003$

(iv) $1.0001 \times 10^{9} = 1000100000$

(v) $5.8 \times 10^{12} = 5800000000000$

(vi) $3.61492 \times 10^{6} = 3614920$

Question 3:Express the number appearing in the following statements in standard form.

(i) 1 micron is equal to $\frac{1}{1000000}$ m.

(ii) Charge of an electron is 0.000, 000, 000, 000, 000, 000, 16 coulomb.

(iii) Size of a bacteria is 0.0000005 m

(iv) Size of a plant cell is 0.00001275 m

(v) Thickness of a thick paper is 0.07 mm

(i) $\frac{1}{1000000} = 1 \times 10^{-6}$

(ii) 0.000, 000, 000, 000, 000, 000, 16 = 1.6×10^{-19}

(iii) $0.0000005 = 5 \times 10^{-7}$

(iv) $0.00001275 = 1.275 \times 10^{-5}$

(v) $0.07 = 7 \times 10^{-2}$

Question 4:In a stack there are 5 books each of thickness 20 mm and 5 paper sheets each of thickness 0.016 mm. What is the total thickness of the stack?

Thickness of each book = 20 mm

Hence, thickness of 5 books = (5 × 20) mm = 100 mm

Thickness of each paper sheet = 0.016 mm

Hence, thickness of 5 paper sheets = (5 × 0.016) mm = 0.080 mm

Total thickness of the stack = Thickness of 5 books + Thickness of 5 paper sheets

= (100 + 0.080) mm

= 100.08 mm

= 1.0008×10^2 mm

What have we discussed

1. Numbers with negative exponents obey the following laws of exponents.

   (a) $a^m \times a^n = a^{m+n}$ (b) $a^m \div a^n = a^{m-n}$ (c) $\left(a^m\right)^n = a^{mn}$

   (d) $a^m \times b^m = (ab)^m$ (e) $a^0 = 1$ (f) $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$

2. Very small numbers can be expressed in standard form using negative exponents.