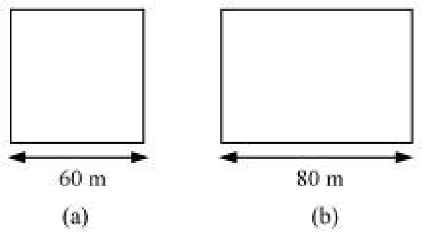
Mensuration

Exercise 10.1

Question 1: A square and a rectangular field with measurements as given in the figure have the same perimeter. Which field has a larger area?



Perimeter of square = 4 (Side of the square) = 4 (60 m) = 240 m

Perimeter of rectangle = 2 (Length + Breadth)

= 2 (80 m + Breadth)

= 160 m + 2 × Breadth

It is given that the perimeter of the square and the rectangle are the same.

160 m + 2 × Breadth = 240 m

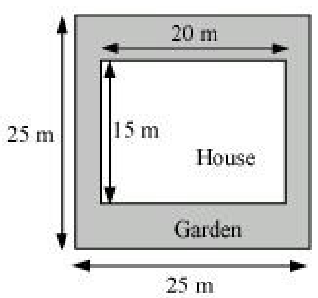
Breadth of the rectangle = $\left(\frac{80}{2}\right)$ m = 40 m

Area of square = $(\text{Side})^2$ = $(60\text{ m})^2$ = 3600 m^2

Area of rectangle = Length × Breadth = $(80 \times 40)\ m^2$ = 3200 m^2

Thus, the area of the square field is larger than the area of the rectangular field.

Question 2: Mrs. Kamran has a square plot with the measurement as shown in the following figure. She wants to construct a house in the middle of the plot. A garden is developed around the house. Find the total cost of developing a garden around the house at the rate of Rs 55 per m^2.



Area of the square plot = $(25\text{ m})^2$ = 625 m^2

Area of the house = (15 m) × (20 m) =300 m^2

Area of the remaining portion = Area of square plot − Area of the house

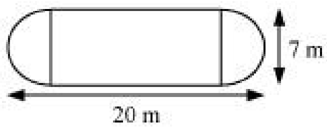
= 625 m^2 − 300 m^2 = 325 m^2

The cost of developing the garden around the house is Rs 55 per m^2.

Total cost of developing the garden of area 325 m^2 = Rs (55 × 325)

= Rs 17,875

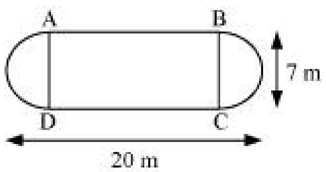
Question 3: The shape of a garden is rectangular in the middle and semi circular at the ends as shown in the diagram. Find the area and the perimeter of the garden [Length of rectangle is 20 − (3.5 + 3.5) metres]



Length of the rectangle = [20 − (3.5 + 3.5)] metres = 13 m

Circumference of 1 semi-circular part = π*r* $= \left(\frac{22}{7} \times 3.5\right)$ m $= 11$ m

Circumference of both semi-circular parts = (2 × 11) m = 22 m



Perimeter of the garden = AB + Length of both semi-circular regions BC and

DA + CD

= 13 m + 22 m + 13 m = 48 m

Area of the garden = Area of rectangle + 2 × Area of two semi-circular regions

$$= \left[(13 \times 7) + 2 \times \frac{1}{2} \times \frac{22}{7} \times (3.5)^2\right] \text{ m}^2$$
$$= (91 + 38.5) \text{ m}^2$$
$$= 129.5 \text{ m}^2$$

Question 4: A flooring tile has the shape of a parallelogram whose base is 24 cm and the corresponding height is 10 cm. How many such tiles are required to cover a floor of area 1080 m2? (If required you can split the tiles in whatever way you want to fill up the corners).

Area of parallelogram = Base × Height

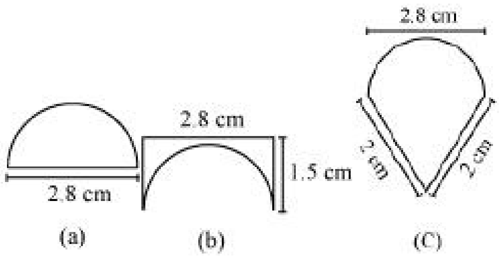
Hence, area of one tile = 24 cm × 10 cm = 240 cm^2

Required number of tiles = $\frac{\text{Area of the floor}}{\text{Area of each tile}}$

$= \frac{1080\ m^2}{240\ cm^2} = \frac{(1080 \times 10000)\ cm^2}{240\ cm^2}$ $(\because 1\ m = 100\ cm)$ = 45000 tiles

Thus, 45000 tiles are required to cover a floor of area 1080 m^2.

Question 5: An ant is moving around a few food pieces of different shapes scattered on the floor. For which food – piece would the ant have to take a longer round? Remember, circumference of a circle can be obtained by using the expression c = 2πr, where r is the radius of the circle.



(a)Radius (r) of semi-circular part = $\left(\frac{2.8}{2}\right)\ cm = 1.4\ cm$

Perimeter of the given figure = 2.8 cm + πr

$= 2.8\ cm + \left(\frac{22}{7} \times 1.4\right)\ cm$

$= 2.8\ cm + 4.4\ cm$

$= 7.2\ cm$

(b)Radius (r) of semi-circular part = $\left(\frac{2.8}{2}\right)\ cm = 1.4\ cm$

Perimeter of the given figure = 1.5 cm + 2.8 cm + 1.5 cm +π (1.4 cm)

$$= 5.8 \text{ cm} + \frac{22}{7}(1.4 \text{ cm})$$
$$= 5.8 \text{ cm} + 4.4 \text{ cm}$$
$$= 10.2 \text{ cm}$$

(c)Radius (r) of semi-circular part = $\left(\frac{2.8}{2}\right) \text{ cm} = 1.4 \text{ cm}$

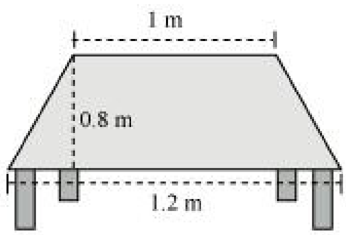
Perimeter of the figure(c) = 2 cm + πr + 2 cm

$$= 4 \text{ cm} + \frac{22}{7} \times (1.4 \text{ cm})$$
$$= 4 \text{ cm} + 4.4 \text{ cm}$$
$$= 8.4 \text{ cm}$$

Thus, the ant will have to take a longer round for the food-piece (b), because the perimeter of the figure given in alternative (b) is the greatest among all.

Exercise 10.2

Question 1: The shape of the top surface of a table is a trapezium. Find its area if its parallel sides are 1 m and 1.2 m and perpendicular distance between them is 0.8 m.



E

Area of trapezium = $\frac{1}{2}$ (Sum of parallel sides) × (Distances between parallel sides)

$$= \left[\frac{1}{2}(1+1.2)(0.8)\right] \text{ m}^2 = 0.88 \text{ m}^2$$

Question 2: The area of a trapezium is 34 cm2 and the length of one of the parallel sides is 10 cm and its height is 4 cm. Find the length of the other parallel side.

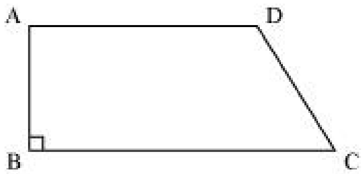
It is given that,area of trapezium = 34 cm^2 and height = 4 cm

Let the length of one parallel side be a. We know that, Area of trapezium = $\frac{1}{2}$ (Sum of parallel sides) × (Distances between parallel sides)

$$34\text{ cm}^2 = \frac{1}{2}(10\text{ cm} + a)\times(4\text{ cm})$$
$$34\text{ cm} = 2(10\text{ cm} + a)$$
$$17\text{ cm} = 10\text{ cm} + a$$
$$a = 17\text{ cm} - 10\text{ cm} = 7\text{ cm}$$

Thus, the length of the other parallel side is 7 cm.

Question 3: Length of the fence of a trapezium shaped field ABCD is 120 m. If BC = 48 m, CD = 17 m and AD = 40 m, find the area of this field. Side AB is perpendicular to the parallel sides AD and BC.



Length of the fence of trapezium ABCD = AB + BC + CD + DA

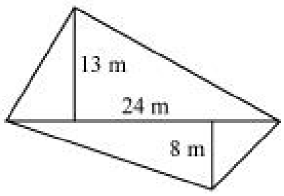
120 m = AB + 48 m + 17 m + 40 m

AB = 120 m − 105 m = 15 m

Area of the field ABCD $= \frac{1}{2}(\text{AD} + \text{BC})\times\text{AB}$

$$= \left[\frac{1}{2}(40+48)\times(15)\right]\text{ m}^2$$
$$= \left(\frac{1}{2}\times 88\times 15\right)\text{ m}^2$$
$$= 660\text{ m}^2$$

Question 4: The diagonal of a quadrilateral shaped field is 24 m and the perpendiculars dropped on it from the remaining opposite vertices are 8 m and 13 m. Find the area of the field.



Length of the diagonal, d = 24 m

Length of the perpendiculars, h_1 and h_2, from the opposite vertices to the diagonal are h_1 = 8 m and h_2 = 13 m

Area of the quadrilateral $= \frac{1}{2} d(h_1 + h_2)$

$= \frac{1}{2}(24\,\text{m}) \times (13\,\text{m} + 8\,\text{cm})$

$= \frac{1}{2}(24\,\text{m})(21\,\text{m})$

$= 252\,\text{m}^2$

Thus, the area of the field is 252 m^2.

Question 5: The diagonals of a rhombus are 7.5 cm and 12 cm. Find its area.

Area of rhombus = $\frac{1}{2}$ (Product of its diagonals)

Therefore, area of the given rhombus

$= \frac{1}{2} \times 7.5\,\text{cm} \times 12\,\text{cm}$

= 45 cm^2

Question 6: Find the area of a rhombus whose side is 6 cm and whose altitude is 4 cm. If one of its diagonals is 8 cm long, find the length of the other diagonal.

Let the length of the other diagonal of the rhombus be x.

A rhombus is a special case of a parallelogram.

The area of a parallelogram is given by the product of its base and height.

Thus, area of the given rhombus = Base × Height = 6 cm × 4 cm = 24 cm^2

Also, area of rhombus = $\frac{1}{2}$ (Product of its diagonals)

$$\Rightarrow 24\text{ cm}^2 = \frac{1}{2}(8\text{ cm} \times x)$$

$$\Rightarrow x = \left(\frac{24 \times 2}{8}\right)\text{ cm} = 6\text{ cm}$$

Thus, the length of the other diagonal of the rhombus is 6 cm.

Question 7: The floor of a building consists of 3000 tiles which are rhombus shaped and each of its diagonals are 45 cm and 30 cm in length. Find the total cost of polishing the floor, if the cost per m2 is Rs 4.

Area of rhombus = $\frac{1}{2}$ (Product of its diagonals)

Area of each tile

$$= \left(\frac{1}{2} \times 45 \times 30\right)\text{ cm}^2$$

= 675 cm^2

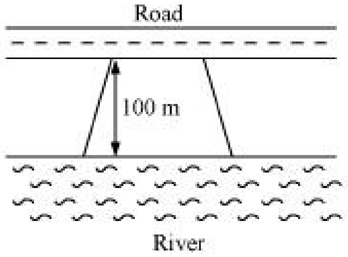
Area of 3000 tiles = (675 × 3000) cm^2 = 2025000 cm^2 = 202.5 m^2

The cost of polishing is Rs 4 per m^2.

Cost of polishing 202.5 m^2 area = Rs (4 × 202.5) = Rs 810

Thus, the cost of polishing the floor is Rs 810.

Question 8: Mehraj wants to buy a trapezium shaped field. Its side along the river is parallel ot and twice the side along the road. If the area of this field is 10500 m2 and the perpendicular distance between the two parallel sides is 100 m, find the length of the side along the river.



Let the length of the field along the road be *l* m. Hence, the length of the field along the river will be 2*l* m.

Area of trapezium = $\frac{1}{2}$ (Sum of parallel sides) (Distance between the parallel sides)

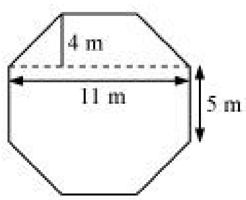
$$\Rightarrow 10500 \text{ m}^2 = \frac{1}{2}(l + 2l) \times (100 \text{ m})$$

$$3l = \left(\frac{2 \times 10500}{100}\right) \text{ m} = 210 \text{ m}$$

$$l = 70 \text{ m}$$

Thus, length of the field along the river = (2 × 70) m = 140 m

Question 9:Top surface of a raised platform is in the shape of a regular octagon as shown in the figure. Find the area of the octagonal surface.



Side of regular octagon = 5 cm

Area of trapezium ABCH = Area of trapezium DEFG

$$\text{Area of trapezium ABCH } = \left[\frac{1}{2}(4)(11+5)\right] \text{ m}^2 = \left(\frac{1}{2} \times 4 \times 16\right) \text{ m}^2 = 32 \text{ m}^2$$

Area of rectangle HGDC = 11 × 5 = 55 m^2

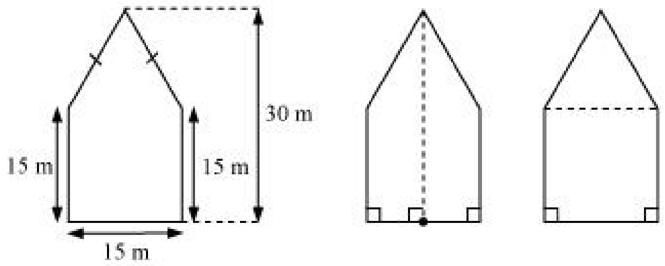
Area of octagon = Area of trapezium ABCH + Area of trapezium DEFG

+ Area of rectangle HGDC

= 32 m^2 + 32 m^2 + 55 m^2 = 119 m^2

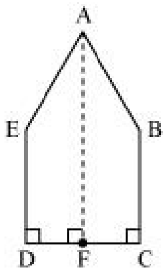
Question 10: There is a pentagonal shaped park as shown in the figure.

For finding its area Asifa and Samina divided it in two different ways.



Find **the area of this park using both ways. Can you suggest some other way of finding its area?**

Asifa's way of finding area is as follows.

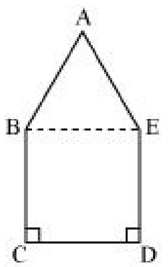


Area of pentagon = 2 (Area of trapezium ABCF)

$$= \left[2 \times \frac{1}{2}(15+30)\left(\frac{15}{2}\right)\right] \text{m}^2$$

= 337.5 m^2

Samina's way of finding area is as follows.



Area of pentagon = Area of ΔABE + Area of square BCDE

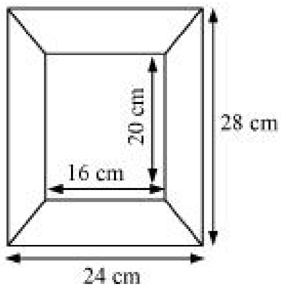
$$= \left[\frac{1}{2} \times 15 \times (30-15) + (15)^2\right] \text{m}^2$$

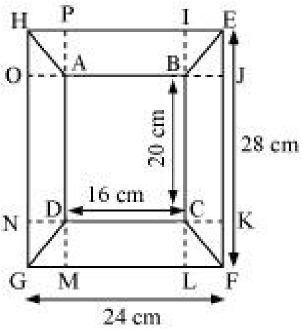
$$= \left(\frac{1}{2} \times 15 \times 15 + 225\right) \text{m}^2$$

$$= (112.5 + 225) \text{ m}^2$$

$$= 337.5 \text{ m}^2$$

Question 11: Diagram of the adjacent picture frame has outer dimensions = 24 cm × 28 cm and inner dimensions 16 cm × 20 cm. Find the area of each section of the frame, if the width of each section is same.





Given that, the width of each section is same. Therefore,

IB = BJ = CK = CL = DM = DN = AO = AP

IL = IB + BC + CL or   28 = IB + 20 + CL

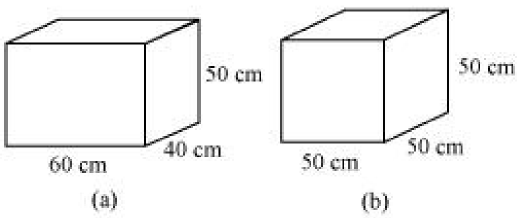
IB + CL = 28 cm − 20 cm = 8 cm

IB = CL = 4 cm

Hence, IB = BJ = CK = CL = DM = DN = AO = AP = 4 cm

Exercise 10.3

Question 1:There are two cuboidal boxes as shown in the adjoining figure. Which box requires the lesser amount of material to make?



We know that,

Total surface area of the cuboid = 2 (lh + bh + lb)

Total surface area of the cube = $6 (l)^2$

Total surface area of cuboid (a) = $[2\{(60) (40) + (40) (50) + (50) (60)\}]$ cm^2

= $[2(2400 + 2000 + 3000)]$ cm^2

= (2×7400) cm^2

= 14800 cm^2

Total surface area of cube (b) = $6 (50 \text{ cm})^2$ = 15000 cm^2

Thus, the cuboidal box (a) will require lesser amount of material.

Question 2: A suitcase with measures 80 cm × 48 cm × 24 cm is to be covered with a tarpaulin cloth. How many metres of tarpaulin of width 96 cm is required to cover 100 such suitcases?

Total surface area of suitcase = $2[(80) (48) + (48) (24) + (24) (80)]$

= $2[3840 + 1152 + 1920]$

= 13824 cm^2

Total surface area of 100 suitcases = (13824×100) cm^2 = 1382400 cm^2

Required tarpaulin = Length × Breadth

1382400 cm^2 = Length × 96 cm

Length = $\left(\frac{1382400}{96}\right)$ cm = 14400 cm = 144 m

Thus, 144 m of tarpaulin is required to cover 100 suitcases. s

Question 3: Find the side of a cube whose surface area is 600 cm2.

Given that, surface area of cube = 600 cm^2

Let the length of each side of cube be l.

Surface area of cube = $6 (\text{Side})^2$

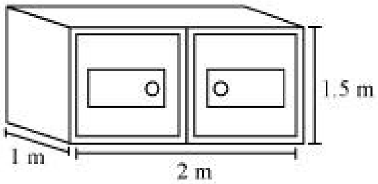
600 cm^2 = $6l^2$

l^2= 100 cm^2

l = 10 cm

Thus, the side of the cube is 10 cm.

Question 4: Rukhsana painted the outside of the cabinet of measure 1 m × 2 m × 1.5 m. How much surface area did she cover if she painted all except the bottom of the cabinet?



Length (l) of the cabinet = 2 m

Breadth (b) of the cabinet = 1 m

Height (h) of the cabinet = 1.5 m

Area of the cabinet that was painted = $2h(l + b) + lb$

= $[2 \times 1.5 \times (2 + 1) + (2)(1)]\ m^2$

= $[3(3) + 2]\ m^2$

= $(9 + 2)\ m^2$

= $11\ m^2$

Question 5: Dawood l is painting the walls and ceiling of a cuboidal hall with length, breadth and height of 15 m, 10 m and 7 m respectively. From each can of paint 100 m2 of area is painted. How many cans of paint will she need to paint the room?

Given that,

Length (l) = 15 m, breadth (b) = 10 m, height (h) = 7 m

Area of the hall to be painted = Area of the wall + Area of the ceiling

= $2h(l + b) + lb$

= $[2(7)(15 + 10) + 15 \times 10]\ m^2$

= $[14(25) + 150]\ m^2$

= $500\ m^2$

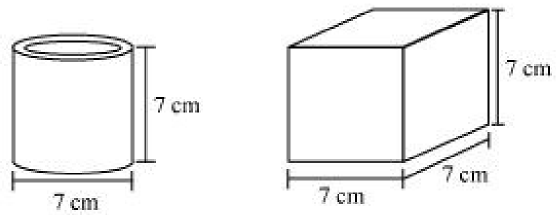
It is given that $100\ m^2$ area can be painted from each can.

Number of cans required to paint an area of $500\ m^2$

$= \frac{500}{100} = 5$

Hence, 5 cans are required to paint the walls and the ceiling of the cuboidal hall.

Question 6: Describe how the two figures at the right are alike and how they are different. Which box has larger lateral surface area?



Similarity between both the figures is that both have the same heights.

The difference between the two figures is that one is a cylinder and the other is a cube.

Lateral surface area of the cube = $4l^2 = 4\ (7\text{ cm})^2 = 196\text{ cm}^2$

Lateral surface area of the cylinder = $2\pi rh = \left(2\times\frac{22}{7}\times\frac{7}{2}\times 7\right)\text{cm}^2 = 154\text{ cm}^2$

Hence, the cube has larger lateral surface area.

Question 7: A closed cylindrical tank of radius 7 m and height 3 m is made from a sheet of metal. How much sheet of metal is required?

Total surface area of cylinder = $2\pi r\ (r + h)$

$$=\left[2\times\frac{22}{7}\times 7(7+3)\right]\text{m}^2$$

$= 440\text{ m}^2$

Thus, 440 m^2 sheet of metal is required.

Question 8: The lateral surface area of a hollow cylinder is 4224 cm2. It is cut along its height and formed a rectangular sheet of width 33 cm. Find the perimeter of rectangular sheet?

A hollow cylinder is cut along its height to form a rectangular sheet.

Area of cylinder = Area of rectangular sheet

$4224\text{ cm}^2 = 33\text{ cm} \times \text{Length}$

$$\text{Length} = \frac{4224\text{ cm}^2}{33\text{ cm}} = 128\text{ cm}$$

Thus, the length of the rectangular sheet is 128 cm.

Perimeter of the rectangular sheet = 2 (Length + Width)

= [2 (128 + 33)] cm

= (2 × 161) cm

= 322 cm

Question 9:A road roller takes 750 complete revolutions to move once over to level a road. Find the area of the road if the diameter of a road roller is 84 cm and length is 1 m.

In one revolution, the roller will cover an area equal to its lateral surface area.

Thus, in 1 revolution, area of the road covered = 2п*rh*

$$= 2 \times \frac{22}{7} \times 42 \text{ cm} \times 1 \text{ m}$$

$$= 2 \times \frac{22}{7} \times \frac{42}{100} \text{ m} \times 1 \text{ m}$$

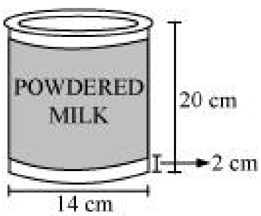
$$= \frac{264}{100} \text{ m}^2$$

In 750 revolutions, area of the road covered

$$= \left(750 \times \frac{264}{100}\right) \text{ m}^2$$

= 1980 m^2

Question 10: A company packages its milk powder in cylindrical container whose base has a diameter of 14 cm and height 20 cm. Company places a label around the surface of the container (as shown in the figure). If the label is placed 2 cm from top and bottom, what is the area of the label.



Height of the label = 20 cm − 2 cm − 2 cm = 16 cm

Radius of the label $= \left(\frac{14}{2}\right) \text{ cm} = 7 \text{ cm}$

Label is in the form of a cylinder having its radius and height as 7 cm and 16 cm.

Area of the label = 2п (Radius) (Height)

$$= \left(2 \times \frac{22}{7} \times 7 \times 16\right) \text{ cm}^2 = 704 \text{ cm}^2$$

Ex. 10.4

Question 1: Given a cylindrical tank, in which situation will you find surface area and in which situation volume.

(a) To find how much it can hold

(b) Number of cement bags required to plaster it

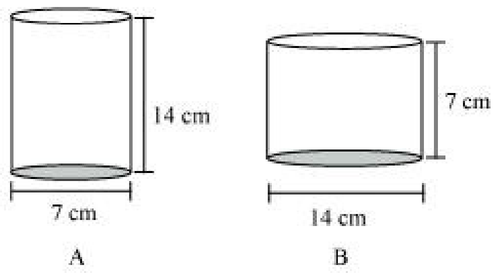
(c) To find the number of smaller tanks that can be filled with water from it.

(a) In this situation, we will find the volume.

(b) In this situation, we will find the surface area.

(c) In this situation, we will find the volume.

Question 2: Diameter of cylinder A is 7 cm, and the height is 14 cm. Diameter of cylinder B is 14 cm and height is 7 cm. Without doing any calculations can you suggest whose volume is greater? Verify it by finding the volume of both the cylinders. Check whether the cylinder with greater volume also has greater surface area?



The heights and diameters of these cylinders A and B are interchanged.

We know that,

Volume of cylinder $= \pi r^2 h$

If measures of *r* and *h* are same, then the cylinder with greater radius will have greater area.

Radius of cylinder A = $\frac{7}{2}$ cm

Radius of cylinder B = $\left(\frac{14}{2}\right)$ cm = 7 cm

As the radius of cylinder B is greater, therefore, the volume of cylinder B will be greater.

Let us verify it by calculating the volume of both the cylinders.

Volume of cylinder A $= \pi r^2 h$

$$= \left(\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 14\right) \text{ cm}^3$$
$$= 539 \text{ cm}^3$$

Volume of cylinder B $= \pi r^2 h$

$$= \left(\frac{22}{7} \times 7 \times 7 \times 7\right) \text{ cm}^3$$
$$= 1078 \text{ cm}^3$$

Volume of cylinder B is greater.

Surface area of cylinder A $= 2\pi r(r+h)$

$$= \left[2 \times \frac{22}{7} \times \frac{7}{2}\left(\frac{7}{2} + 14\right)\right] \text{ cm}^2$$
$$= \left[22 \times \left(\frac{7+28}{2}\right)\right] \text{ cm}^2$$
$$= \left(22 \times \frac{35}{2}\right) \text{ cm}^2$$
$$= 385 \text{ cm}^2$$

Surface area of cylinder B $= 2\pi r(r+h)$

$$= \left[2 \times \frac{22}{7} \times 7 \times (7+7)\right] \text{ cm}^2$$
$$= (44 \times 14) \text{ cm}^2$$
$$= 616 \text{ cm}^2$$

Thus, the surface area of cylinder B is also greater than the surface area of cylinder A.

Question 3: Find the height of a cuboid whose base area is 180 cm2 and volume is 900 cm3?

Base area of the cuboid = Length × Breadth = 180 cm^2

Volume of cuboid = Length × Breadth × Height

900 cm^3 = 180 cm^2 × Height

$$\text{Height} = \left(\frac{900}{180}\right) \text{cm} = 5 \text{ cm}$$

Thus, the height of the cuboid is 5 cm.

Question 4: A cuboid is of dimensions 60 cm × 54 cm × 30 cm. How many small cubes with side 6 cm can be placed in the given cuboid?

Volume of cuboid = 60 cm × 54 cm × 30 cm = 97200 cm^3

Side of the cube = 6 cm

Volume of the cube = $(6)^3$ cm^3 = 216 cm^3

Required number of cubes = $\frac{\text{Volume of the cuboid}}{\text{Volume of the cube}}$

$$= \frac{97200}{216} = 450$$

Thus, 450 cubes can be placed in the given cuboid.

Question 5:Find the height of the cylinder whose volume is 1.54 m3 and diameter of the base is 140 cm?

Diameter of the base = 140 cm

Radius (r) of the base $= \left(\frac{140}{2}\right) \text{cm} = 70 \text{ cm} = \frac{70}{100} \text{m}$

Volume of cylinder $= \pi r^2 h$

$$1.54 \text{ m}^3 = \frac{22}{7} \times \frac{70}{100} \text{ m} \times \frac{70}{100} \text{ m} \times h$$
$$h = \left(\frac{1.54 \times 100}{22 \times 7}\right) \text{m} = 1 \text{ m}$$

Thus, the height of the cylinder is 1 m.

Question 6: A milk tank is in the form of cylinder whose radius is 1.5 m and length is 7 m. Find the quantity of milk in litres that can be stored in the tank?

Radius of cylinder = 1.5 m

Length of cylinder = 7 m

Volume of cylinder $= \pi r^2 h$

$= \left(\frac{22}{7} \times 1.5 \times 1.5 \times 7 \right) \text{ m}^3$

$= 49.5 \text{ m}^3$

$1\text{m}^3 = 1000$ L

Required quantity = (49.5 × 1000) L = 49500 L

Therefore, 49500 L of milk can be stored in the tank.

Question 7: If each edge of a cube is doubled,

(i) how many times will its surface area increase?

(ii) how many times will its volume increase?

(i) Let initially the edge of the cube be l.

Initial surface area = $6l^2$

If each edge of the cube is doubled, then it becomes $2l$.

New surface area = $6(2l)^2 = 24l^2 = 4 \times 6l^2$

Clearly, the surface area will be increased by 4 times.

Question 8: Water is pouring into a cubiodal reservoir at the rate of 60 litres per minute. If the volume of reservoir is 108 m3, find the number of hours it will take to fill the reservoir.

Volume of cuboidal reservoir = 108 m^3 = (108 × 1000) L = 108000 L

It is given that water is being poured at the rate of 60 L per minute.

That is, (60 × 60) L = 3600 L per hour

Required number of hours $= \frac{108000}{3600}$ = 30 hours

Thus, it will take 30 hours to fill the reservoir.

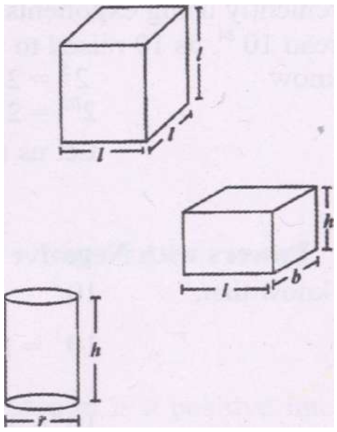
What have we Discussed

1. Area of
   (i) a trapezium = half of the sum of the lengths of parallel sides x perpendicular distance between them.
   (ii) a rhombus = half the product of its diagonals.
2. **Surface area** of a solid is the sum of the areas of its faces.
3. Surface area of

   a cuboid = 2**(lb + bh + hi)**

   a cube = $6 / ^{2}$

   a cylinder = **2vr(r + h)**



4. Amount of region occupied by a solid is called its **volume.**
5. Volume of

   a cuboid = / x **b** x **h**

   a cube = 1^3

   a cylinder **= nr^2 h**

   (i) $1\ cm^3 = 1\ mL$

   (ii) $1L = 1000\ cm^3$

   (iii)$1\ m^3 = 1000000\ cm^3 = 1000L$